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P4_8 The Solar Escapee

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Abstract

The paper investigates the circumstances in which a planet with an orbital radius of 1 AU from the Sun would escape the solar system due to the force supplied by the solar radiation pressure of the Sun. It is found that Earth, while fixing the mass, would require a radius above 3.06×10^{26} m before it would be ejected from the solar system. Alternatively a planet with the same density and orbit as Earth, but an unfixed mass, would require a radius below 2.76×10^{-7} m.

Introduction

Objects throughout the solar system experience solar radiation pressure from exposure to the Sun. The forces due to these pressures are extremely small, and the effects are only apparent on small bodies such as asteroids due to their low mass. However, the concept of a pressure is such that the force it produces increases with the size of surface area the radiation can interact with. Solar-sail spacecraft are based around this concept. If a large planet was to exist close to the Sun with a low mass, the magnitude of the force from the radiation pressure could be sufficient to significantly alter the planets orbit.

This paper applies the concept of solar radiation pressure to discover the alterations to Earth required which would enable its escape from the solar system.

Solar Radiation Pressure

Maxwell's theory of electromagnetism states that electromagnetic waves carry a momentum described by Planck's constant divided by its wavelength. There are three processes in which radiation can impart a change of momentum onto an object: absorption, reflection and emission. Emission of thermal radiation from Earth imparts an isotropic change in momentum, resulting in no net change of momentum and so it is ignored. The equation which quantifies radiation pressure P_r for radiation intensity I is

$$P_r = \frac{I}{c}, \qquad \text{(Eq. 1)}$$

where $c = 3\times10^{8}$ ms^{-1} is the speed of light. Equation 1 describes all three of the processes previously mentioned. The only alteration to Equation 1 is where the pressure is twice the value for reflected radiation, due to the change in momentum being doubled. The amount of radiation that the whole of the Earth's surface reflects on average is 30% of all incoming radiation; the remaining incident radiation is absorbed. The intensity of incident radiation from the sun can be found using the equation

$$I = \frac{L_S}{4\pi r^2}, \qquad \text{[2](Eq.2)}$$

where r is the orbital radius and $L_S = 3.9\times10^{26}$ W is the solar luminosity [2]. The total radiation pressure P_T incident on Earth is therefore

$$P_T = \frac{1}{c}(0.7I + 2\times 0.3I) = \frac{1.3L_S}{4\pi r^2 c}. \qquad \text{(Eq. 3)}$$

Once the radiation pressure has been obtained the total radiation force on the planet, $\underline{F_{RT}}$, can be found by the relationship

$$\underline{F_{RT}} = P_T A\hat{n}, \qquad \text{(Eq. 4)}$$

where A is the surface area the radiation is incident upon, and $\hat{n}$ is the vector normal to the area. As 1 AU is sufficiently far from the Sun, all angular dependence of the surface is ignored and the area can therefore be taken as the cross-sectional area $A = \pi R^2$ with a planetary radius R. The vector $\hat{n}$ is then in the same direction as $\hat{r}$, radially away from the Sun.

Escaping the solar system

It is possible that a planet with a high velocity can escape the gravitational confinement of the

Sun. To discover if the radiation pressure provides a sufficient force to continuously overcome gravity we must compare the orbital velocity v_o with the escape velocity v_e. The force due to gravity F_g is described by

$$\underline{F_g} = -\frac{GM_S m_e}{r^2}\,\hat{r}\,. \qquad \text{(Eq. 5)}$$

where G = 6.67×10^{-11} m^{3}kg^{-1}s^{-2} is the gravitational constant, M_s = 1.99×10^{30} kg is the solar mass and m_e = 5.97×10^{24} kg is the Earth's mass [3]. The net force $\underline{F_N}$ acting on Earth is then

$$\underline{F_N} = \underline{F_g} - \underline{F_{RT}} = \underline{F_g}(1-\beta)\,, \qquad \text{(Eq. 6)}$$

where β is defined as

$$\beta = \frac{F_{RT}}{F_g} = \frac{1.3R^2 L_S}{4cGM_S m_e}\,. \qquad \text{(Eq. 7-1)}$$

From Eq. 6 it is the clear that if β is greater than 1, then $\underline{F_{RT}}$ dominates, resulting in the escape of stationary object from the solar system. However for an orbiting object additional considerations need to be made. It is important to note that β is r independent, and therefore will remain constant as a planet escapes. When β is found, Eq. 7-1 finds the required R for escape assuming m_e remains constant (but changes in density, $\rho = \frac{m_e}{\frac{4}{3}\pi R^3}$). Alternatively the size of a generic planet, orbiting at 1 AU with the same average density as Earth of ρ = 5.51×10^3 kgm^{-3}, which would escape the solar system could be found. This is achieved by using the density equation with Eq. 7-1 to get

$$\beta = \frac{1.3L_S}{\frac{16}{3}cGM_S\rho\pi R}\,. \qquad \text{(Eq. 7-2)}$$

An orbiting object has an orbital velocity found by equating the gravitational force to the centripetal force, resulting in

$$v_o^2 = \frac{GM_S}{r}\,, \qquad \text{(Eq. 8)}$$

which considers an orbit in which radiation pressure is not significant. This is contrasted to the escape velocity, which can be found by considering the energy E_e required to move Earth to infinity using the net force in Eq. 6:

$$E_e = \int_r^\infty \underline{F_N}\cdot d\underline{r} = \frac{(1-\beta)GM_S m_e}{r}\,. \qquad \text{(Eq. 9)}$$

Eq. 9 equals the kinetic energy Earth requires to escape, therefore its velocity is found as

$$v_e^2 = \frac{2GM_S}{r}(1-\beta)\,. \qquad \text{(Eq. 10)}$$

If the orbital velocity of Earth is greater than the velocity required for it to escape, due to a significant radiation pressure force, then it will escape. Instead, if the orbital velocity is less than the escape velocity, the planet will remain confined to the Sun's orbit; however the orbit would change to a higher orbit to account for the significant radiation pressure. Therefore:

$$v_o^2 \geq v_e^2\,,$$
$$\frac{GM}{r} \geq \frac{2GM}{r}(1-\beta)\,,$$
$$\beta \geq \frac{1}{2}\,.$$

Alterations to Earth

With a condition for β now calculated, the minimum required radius for Earth to escape the solar system due to radiation pressure can be found from Eq. 7-1 as R = 3.06×10^{13} m, 5×10^6 times larger than its current radius [3]. If Earth had this radius, its density would be ρ = 5×10^{-17} kgm^{-3} which is likely smaller than the required density to ensure the planet is gravitationally bound together. This could be further investigated in the future by considering the Jean's mass of this altered Earth. If a different planet was located in place of Earth, at 1 AU orbit, but with the same density of Earth, Eq. 7-2 declares that for this planet to escape the solar system it would need a radius below 2.76×10^{-7} m. This radius is approximately the size of a dust particle [4].

Conclusion

The force provided by the solar radiation appears very small. Due to the extremes of planetary parameters calculated it would seem impossible for a planet to exist that could be ejected from our solar system via radiation pressure. A state in which Earth has a radius of 3.06×10^{26} m also has an extremely low density. Alternatively a high density planet will only be ejected if it is smaller than a dust particle and therefore has a very tiny mass.

References

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