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P2_4 Tea Saboteur

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Abstract

Practical jokes in the university lifestyle are unfortunately almost unavoidable. However, "If you can't beat 'em, join 'em" and in that spirit, this paper assesses the feasibility of cooling your housemate's cup of tea down with ice. It is calculated that it would take approximately 16.5 minutes for a sphere of ice of radius 1.5cm to completely melt in a cup of tea that was initially 60°C.

Introduction

Practical jokes are often regarded as a somewhat inescapable formality of student life. As such, inspired by 2011 A3_10 'Fancy a cuppa?' [1], this paper examines the possibility of destroying your flatmate's morning brew, and getting away with it. The paper investigates the time it would take for an 'ice-sphere' of radius 1.5cm to completely melt in a cup of tea which was initially at an optimum drinking temperature of 60°C [2]. By integrating the temperature gradient over a spherical surface we determine whether or not you could escape as a successful tea saboteur.

Theory

To obtain a value for the time it takes for the ice to fully melt and cool the tea we consider heat transfer by thermal conduction [3],

$$\frac{dQ_I}{dt} = -\kappa A \frac{dT}{dr}, \qquad (1)$$

where dQ/dt is the rate of heat flow into the ice which causes it to melt, κ is the thermal conductivity of the tea, A is surface area over which we integrate, and dT/dr is the temperature gradient throughout the ice-tea system. Conventionally, ice that is used to cool drinks is cube shaped, hence the term 'ice-cube', however for symmetric simplicity we consider our 'ice-cube' to be spherical, and furthermore, assume the cup of tea as also spherical, (to a good approximation). We can therefore use a spherical co-ordinate system

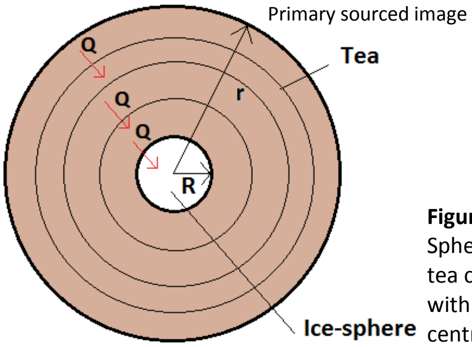

Figure 1: Spherical tea cup with ice at centre

to address our problem. Figure 1 illustrates our set-up. We assume that the ice is located at the centre of the tea, at the origin, and sits there freely irrespective of any buoyancy force. As seen, the ice-sphere has a radius *R*, and *r* denotes the radial distance outwards.

We assume the ice-tea system rapidly approaches thermal equilibrium. This is such that the temperature gradient dT/dr is time independent, and the heat flow throughout the tea and into the ice is constant [3] (as illustrated), hence $dQ/dt = C$. Furthermore, the heat energy required to melt the ice is given by $L \times m$ [3] where L is the latent heat of fusion, and m is the mass. Therefore equation (1) can be rewritten as;

$$\frac{dQ_I}{dt} = L\frac{dm}{dt} = -\kappa A \frac{dT}{dr} = C. \qquad (2)$$

Whilst assuming the system approaches equilibrium rapidly, may seem quite severe, we consider the approximation to be reasonable, as discussed later.

By integrating dT/dr over a series of concentric spherical shells with area $A = 4\pi r^2$ we can obtain the constant *C*. Then, using the equivalence $L\,dm/dt = C$ we can identify how long it takes for the mass of ice to melt. Cooling due to convection is not considered within this paper, as we only require the heat change across the ice-tea interface to determine the melting time.

Substituting the area A into equation (2) and integrating gives,

$$T(r) = \frac{C}{4\pi r\kappa} + c. \quad (3)$$

From equation (3) we can see that as $r \to \infty$, the integration constant c must equal T_{Tea}. At the edge of the cup, the temperature of the tea would remain approximately unchanged; and at an infinite distance away from the ice, the temperature would also be unchanged, $T(\infty) = T_{Tea}$. Therefore, due to this parallel, we can model the cup as infinitely large. Additionally, at $r = R$, T equals the temperature of the ice, T_{ice}. As such, equation (3) becomes,

$$T_{ice} = \frac{C}{4\pi R\kappa} + T_{Tea}, \quad (4)$$

and rearranging for the constant C gives,

$$C = (T_{ice} - T_{Tea})4\pi R\kappa. \quad (5)$$

Having obtained the constant C, we can use this in equation (2). Simultaneously we want the mass in terms of a radius, so we only have one variable, hence equation (2) can written as,

$$L\rho 4\pi R^2 \frac{dR}{dt} = (T_{ice} - T_{Tea})4\pi R\kappa, \quad (6)$$

where ρ is the density of ice. Separating the variables and integrating results in;

$$\frac{R^2}{2} = \frac{(T_{ice} - T_{Tea})\kappa t}{L\rho} + c. \quad (7)$$

To remove the constant of integration we apply known boundary conditions. At time $t = 0$, the radius of the ice is unchanged, hence $R = R_0$. Substituting these conditions in, we see $c = {R_0}^2/2$. Therefore as the radius R diminishes as the ice melts, at R = 0, equation (7) finally becomes,

$$t(R = 0) = -\frac{{R_0}^2 L\rho}{2(T_{ice} - T_{Tea})\kappa}. \quad (8)$$

Results and Discussion

In this particular scenario, an ice-sphere of 1.5cm radius is the weapon of choice. We assume that that the ice-sphere is initially 0°C and has a density equal to 916.2 kg/m [4]. The latent heat of fusion is 333.5 kJ/kg [3], and the practical joke involves reducing the temperature of the tea from 60°C. The thermal conductivity of water, κ is equal to 5.8×10^{-4}kJ/s·m·K [5]. Using these values and equation (8) we calculate it would 988s (16.5 minutes) for the ice to completely melt.

As previously mentioned, this entire argument is based on the assumption that the tea and ice are only ever in thermal equilibrium. In reality of course, as the ice is introduced to the tea, the ice would start to melt before it reached thermal equilibrium with the tea. We have assumed that the amount it melts by is negligible however, as we assume the energy involved in changing the state of the ice, $Q = mL$, entirely dominates the energy involved in cooling the tea down over the temperature range, $Q = mc\Delta T$. As such, our calculation is only approximate, yet as the energy associated with mL would be greater than $mc\Delta T$, we are confident that this assumption is reasonable. Furthermore, the tea would also cool due to radiation in a manner described by Newton's law of cooling [6] which would alter the time it takes to cool. As a suggestion for future work, an investigation into the contribution of these factors would complement our analysis.

Conclusion

The model predicts that the time required for ice of radius 1.5cm to melt and cool the cuppa is approximately 988s. According to Northumbria University, the ideal brewing time is 6 minutes [2]. Based upon our result, this would therefore allow plenty of get-away time and even leave you enough time to steal your housemate's digestives too; although that might be classed as taking the biscuit.

References

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