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# P2_5 Bouncing Back

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**Abstract**
This paper aims to calculate the probability of a person being reflected back from a cliffs edge whilst try to run off of it. This is by treating the cliff as a negative potential step, and the person as a point of mass. The probability of reflection is found to be $R^2$ = $e^{-2.5\text{x}10^{37}}$ which although extremely small, is non-zero. The paper also discusses the limits of the model presented when considering decoherence.

**Introduction**
To explain phenomena that occur at a quantum mechanical scale, it is often useful to use macroscopic analogies to better illustrate what is happening. However this scaling up can create counter intuitive events, that classically are impossible. This paper discusses the probability of a person being reflected back from the edge of a cliff whilst trying to run off by treating it as a negative potential step.

**Theory**
In quantum mechanics, when a wave packet is incident on a potential step there is a possibility that it will either be transmitted through the step or reflected back, as shown in figure 1.

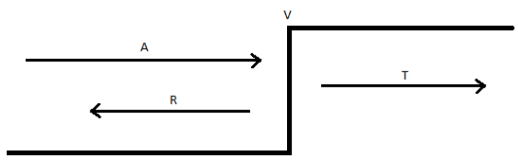


Figure 1. Diagram of potential step

Where $A$ is the wave packet travelling with energy $E$, $R$ is the reflected wave, $T$ is transmitted, and $V$ is the size of the potential step. In theory these possibilities also exist for a negative potential step, which is the main focus of this paper.

The equation for the probability of reflection for a standard, positive potential well is

$$R^2 = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)^2 \quad , \tag{1}$$

where $k_1 = \frac{\sqrt{2mE}}{\hbar}$, $k_2 = \frac{\sqrt{2m(E-V)}}{\hbar}$, and $R^2$ is the probability of reflection [1].

This treats the potential step as a sheer step with no gradient at all. This will give incorrect results when scaled up, as real situations cannot be completely sheer, as such we will need to consider a soft step for this problem. A soft step is a gradual change in potential as opposed to an immediate one. The reflection probability of this kind of step is described in *P.L. Garrido et al. 2011* [2]. This paper also provides an equation for the probability of reflection from a soft step by treating the step as a curve.

$$R^2 = \left(\frac{\sinh(\frac{\pi}{2}(k_2 - k_1)L)}{\sinh(\frac{\pi}{2}(k_2 + k_1)L)}\right)^2 \tag{2}$$

Where $L$ is the length of the sloping potential. So for a completely flat surface $L = \infty$, and for a sheer drop $L = 0$. Taking $\sinh(x) = \frac{e^x - e^{-x}}{2}$, $a = \frac{\pi}{2}(k_2 - k_1)L$, and $b = \frac{\pi}{2}(k_2 + k_1)L$ to simplify the equation we get

$$R^2 = \left(\frac{e^a - e^{-a}}{e^b - e^{-b}}\right)^2 \ . \tag{3}$$

**Calculation**

To find the macroscopic chances of reflection from a negative potential step we shall use the gravitational potential energy as the step potential *V*

$$V=-mgh \qquad (4)$$

the kinetic energy shall be the energy of the incident wave packet *E.*

$$E = \frac{1}{2}mv^2 \qquad (5)$$

Where *m* is mass, *g* is acceleration due to gravity, *h* is cliff height and *v* is velocity.

Taking Beachy Head, the famous suicide hotspot in Sussex, as the potential step: *h*=162m [3], the cliff is nearly sheer but not completely so we will use *L*=1 for simplicity, *g*=9.81ms$^{-2}$, the mass of a person as *m*=70kg, and the velocity of a person running as *v*=6ms$^{-1}$. Using these figures we get *V*=111245.4J, and *E*=1260J from equations (4) and (5) respectively.

Using these values for *E* and *V*, we can find $k_1$and $k_2$, and therefore *a* and *b, g*iving: *a* = 5.3x10$^{37}$ and *b* = 6.55x10$^{37}$.

Then using equation (3) we get

$$R^2 = \left(\frac{e^{5.3\text{x}10^{37}} - e^{-5.3\text{x}10^{37}}}{e^{6.55\text{x}10^{37}} - e^{-6.55\text{x}10^{37}}}\right)^2$$

We can see that $e^{-5.3\text{x}10^{37}}$and $e^{-6.55\text{x}10^{37}}$ are extremely small and so tend to 0, leaving

$$R^2 = \left(\frac{e^{5.3\text{x}10^{37}}}{e^{6.55\text{x}10^{37}}}\right)^2$$

This gives the probability of reflection, $R^2$, as $e^{-2.5\text{x}10^{37}}$.

**Discussion**

From the probability of reflection found above, we can see that while it is almost certain a person running off a cliff will not be reflected back on to solid ground, the probability is non-zero and so the possibility does remain.

In practice this probability will be even less. This paper uses a very simple model, comparing a person to a single particle or wave packet, and a cliff as a potential step. In reality the person is vast multitude of particles and the cliff a much more complicated potential than the one presented. This problem of complexity when scaling up quantum mechanics is known as decoherence [4]. This phenomenon effectively prohibits quantum mechanical effects at the scale discussed in the paper, though the probability value will still be non-zero.

**References**

[1]Paul. A. Tipler and Gene Mosca, *Physics for Scientists and Engineers,* Sixth Edition*,* p.1212.

[2]P.L. Garrido et al *, Paradoxical reflection in quantum mechanics,* American Journal of Physics, Volume 79, Issue 12, pp. 1218-1231 (2011).

[3]http://www.eastbourne.org/tourism/beachyhead/ accessed on 30/10/2013.

[4]A.I.M. Rae, *Quantum Mechanics*, Fifth edition, p. 309.