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P2_3 Tennis Ball Tunnelling

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Abstract

This paper aims to calculate the probability of a tennis ball quantum tunnelling through a tennis racket. This is done by treating the tennis ball as a single particle, and making the potential barrier equal to the energy required to break the strings classically. The probability of tunnelling through a racket is found to be 3.6$e^{-2.9\times10^{\wedge}31}$. This very low probability matched with what would be expected. The paper also briefly discusses the problem of decoherence when applying quantum mechanics to macroscopic systems.

Introduction

Often, when explaining the phenomena that occur in quantum mechanics to a layman, it is simpler to use macroscopic analogies. For example, explaining quantum mechanical tunnelling by suggesting that it is possible, however unlikely, that a tennis ball will tunnel though the strings of a racket instead of simply bouncing off. This paper will try and quantify the probability of this occurring.

Theory

When a beam of particles are incident on a potential barrier the majority of particles will do as expected and be reflected, however, some particles will be transmitted out of the other side of the barrier; they will have tunnelled through. This is quantum tunnelling. The following equation describes the probability of this tunnelling event [1],

$$\frac{|F|^2}{|A|^2} = \frac{16E(V_0 - E)}{V_0^2}\exp(-2\kappa b) \qquad (1)$$

where $\frac{|F|^2}{|A|^2}$ is the tunnelling probability, E is the energy of incident particles, V_0 is the size of the potential barrier, b is the width of the barrier, and κ is

$$\kappa = [2m(V_0 - E)/\hbar^2]^{1/2} \qquad (2)$$

m being the mass of the particles and $\hbar$ the reduced Planck constant.

In this paper, E will be the kinetic energy of the tennis ball, $E = \frac{1}{2}mv^2$, and the potential barrier will be taken as the energy needed to snap the strings classically. If the material breaks at, or just after, its limit of proportionality this can be found using the Young's Modulus of the strings.

$$Y = \frac{Stress}{Strain} = \frac{\sigma}{\varepsilon} \qquad (3)$$

Where Y is the Young's Modulus, the stress is $\frac{F}{A}$, where F is the force applied, and A is the cross-sectional area of the material. The strain is $\frac{\Delta x}{x}$, where x is the length of wire before stretching and Δx is the distance the wire stretches. If the breaking stress of the wire , σ_b, is used in equation (3), the breaking strain can be found. As the initial length of the wire is known, the amount the string needs to be stretched before it breaks, Δx_b, can be found.

$$\Delta x_b = \frac{\sigma_b x}{Y} \qquad (4)$$

Calculation

Most strings on tennis rackets are made of nylon [2], which has a breaking stress of $75\text{x}10^6$ $\mathrm{N/m^2}$ and Young's Modulus of ~$3\text{x}10^9$ $\mathrm{N/m^2}$ [3]. Nylon's breaking stress is actually beyond the limit of proportionality but for this paper it will be treated as though it is within to simplify the problem. Due to the orders of magnitude involved in the calculations it should not significantly alter the results. For simplicity, we will assume the face of a tennis racket is a circle with a diameter of x=0.7m. Putting these numbers into equation (4) we get Δx_b as 0.0175m.

Using this and rearranging the Young's Modulus equation (3) with the breaking strain, and converting the force, F, to energy, V_0, using the work done equation, we get

$$V_0 = A(\sigma_b)\,\Delta x_b. \qquad (5)$$

The average mass of a tennis ball is 57g [4]. The diameter of each string is 1.3mm [2] giving an area of 1.3x10^{-6}m^2, where the diameter of the string is also its width. A tennis ball will strike several strings at once; we shall let this be 20, meaning $A = 1.3 \times 10^{-6} \times 20$. The speed of the tennis ball will be 100kmph, or ~28m/s.

Putting these numbers into equations (5), (2), and (1) gives a probability of $3.6e^{-2.9\times10\wedge31}$ for the tennis ball tunnelling through the racket. This probability is so low it may as well be 0; this matches with what would be expected from theory.

Discussion

From the probability above, it is shown that although theoretically still possible, the chance of a tennis ball quantum tunnelling through a racket is effectively nil.

In this paper we have treated the tennis ball as a single particle, and the racket strings simply as a potential barrier. In reality both of these objects are made up of an enormous number of particles, and so the actual likelihood of the tennis ball tunnelling are in fact much smaller. The number of particles involved, and therefore the number of wave functions would make a full computation of the probabilities impossible. This difference in complexity between the macroscopic and quantum scales is known as decoherence [5], and it effectively forbids the study of quantum interference patterns produced in the macroscopic world. This is the reason for the simple model used in the paper.

References

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