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P6_3 One Hundred and Eighty!

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Abstract

This paper uses a computer simulation to examine how darts players of different accuracies may increase their average score by aiming for different targets on the board. Although it is hard to say whether the model of accuracy used closely matches that of real players, the program does indicate that for high accuracy the treble twenty is the best target, and for less accurate players the treble nineteen or elsewhere in the lower left quadrant may present a better target and increase their average score, despite the maximum possible score in this region being lower.

Virtual Dartboard

This experiment is a computer simulation of darts players of various accuracies, and aims to find the optimum target for each player as a function of accuracy. It has been conducted via a simulation of nearly one hundred billion throws in a self-constructed C program. The program generates a virtual dart board comprised of more than 30,000 individual points, arranged radially on a series of rings, each separated by $\frac{1}{100}$ of the radius of the board, R_B, upon which all the points are also $\frac{R_B}{100}$ apart. Each point falls into a discrete bin as shown in Figure 1, each with an associated score. The thin outer ring in Figure 1 is worth double the nominal score, and the thin ring at half the radius of the board is worth triple the nominal score. The centre of the board is worth 50 points, and the area surrounding the centre is worth 25 [1].

Virtual Players

To begin we need to define a player's accuracy. A player with zero accuracy is defined as having no control over where his or her dart lands, and a player with infinite accuracy has complete control - wherever the player aims, the dart lands. Between these extremes we model accuracy η by defining the probability of a dart hitting the board at any point as proportional to $\frac{1}{(\eta d)^2}$, where d is the distance between the point at which the dart lands and the target the player aimed for. Figure 2 shows that as η increases, the probability of a hit moves from a uniform spread to a delta function. The probabilities are all capped at unity. Although the integrals of these probability functions are not equal, this is normalised within the program. The program simulates sixty players each of increasing accuracy (for clarity only 15 of the 60 are shown in Figure 2). It may be useful for other groups to map the actual distribution of hits for real players, as the distribution described here may not match the true distribution. This would involve a study of a great deal of players and as such could not be conducted for this paper.

Procedure of Program

For each player (and as such each accuracy) the program cycles through all 30,000 points on the board, calling each one in turn the target. For each specified target the program then finds the average score the player can expect to gain aiming for that target. It does this by again cycling through all 30,000 points, finding the distance between each one and the target point, and as such the probability of a dart landing there. This probability is then multiplied by the score associated with that point, and added to a running total. Once all points have been tested and the contributions have been summed, this running total is assigned to the target as the expected score. The next target is then selected and is assigned an expected score by the same process, and this repeats for all possible targets. The target with the best expected score is then returned for this player. This repeats for all 60 players until each has a target with the best expected score assigned to them. These targets for each player (and each level of accuracy) are plotted in Figure 1. The lower accuracies are the points nearest the centre of the board.

Discussion of Results

As expected when accuracy is high and the spread of hits is almost a delta function, the best target is the treble twenty, however an actual delta function of hits is impossible due to the non-zero diameter of a dart. It is of course possible to fit three darts in the treble twenty section of the board, provided the darts do indeed land slightly separated, resulting in a score of one hundred and eighty. When the spread of hits on the board is completely uniform over a circle of radius R_B the best target is the centre of the board, as no darts miss the board in this case. Between these extremes the target moves around the board, favouring areas in which high-scoring numbers are grouped. It seems counter-intuitive that in some cases aiming for lower numbers will generate higher average scores. The program shows us that a quarter of the players simulated scored higher when aiming for seven than for sixty (treble twenty). The plot in the left panel of Figure 1 may be of great use to players who find that aiming for large numbers is not appropriate for their level of accuracy.

It is worth noting that the transition from the lower left to the treble twenty is not smooth, but is a distinct step, and there is another step between the centre of the sixteen and the lower edge of the sixteen, as highlighted in Figure 1. We propose these jumps are due to the multi-peaked structure of the board. The targets can become trapped on a peak, surrounded by lower scores. In such an instance, for the players' score to increase the target cannot move smoothly, else it would move into these regions of lower scores, but must instead jump to another higher peak. This could be an avenue for further study.

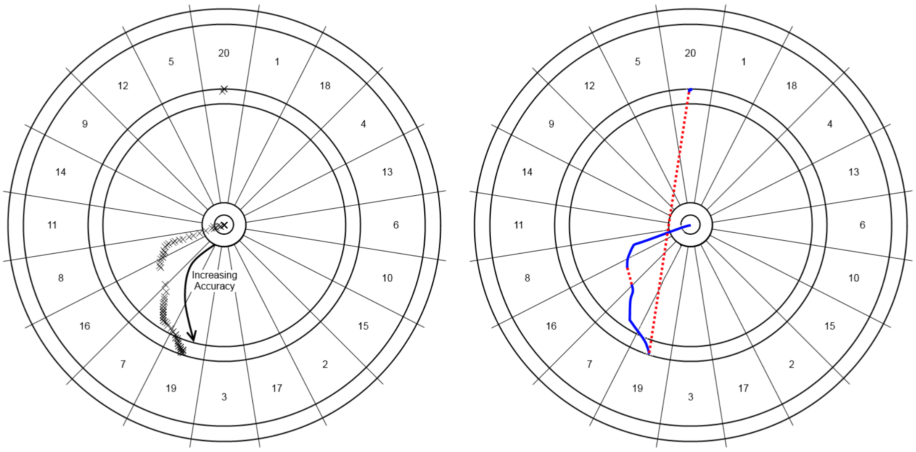

Fig. 1: (Left) best target as a function of player accuracy and (right) points shown as a smooth function (blue) with discontinuities highlighted(red, dashed).

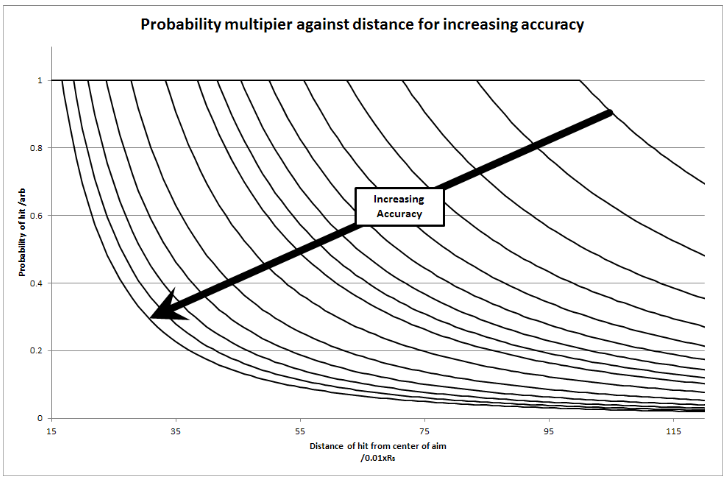

Fig. 2: Probability of hit as a function of player accuracy and distance from target.

References

[1] http://www.a180.co.uk/shopimages/products/extras/Unicorn%20Eclipse%20Pro%20Dartboard%20800%20x%20800.jpg accessed 11/10/2013.