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P6_2 Theoretical Limit to the Depth at Which a Snorkel May Be Used

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Abstract
Using simple mechanics and basic information on the human body we impose various limits on the length of a snorkel used for breathing underwater. The volume of a human lung and our need for oxygen imposes a limit of $3.4l$ on the volume of the snorkel, roughly corresponding to a length of $10.8m$. The buoyancy force of a snorkel imposes a limit of $2.7m$ to prevent constant floating, and the limit imposed by the strength of a plastic tube is greater than the depth to which any human has ever dived, and is ignored.

Introduction

There are many factors which may impose physical limits on the use of a snorkel as an underwater breathing apparatus for humans. The most obvious of these is the total internal volume of the snorkel, which must not exceed the total volume of a humans lungs. In fact it must be considerably less than this, as the difference between these volumes gives the total amount of new air the lungs can take in with every breath (stroke). Another limit will be imposed by the construction of the snorkel and its ability to withstand the water pressure at lower depths. The final limit this paper considers is the one imposed by the additional buoyancy a snorkel generates when filled with air, which will eventually make diving with long snorkels impossible.

In this paper we apply some simple calculations to measure these limitations and find which process gives the upper boundary for the maximum length a snorkel can be.

Oxygen Requirements

In order to survive, a human inhales air with an oxygen concentration of 21% and exhales air with only 16% oxygen [1]. This implies that even if as humans we can successfully absorb 100% of the oxygen we inhale we would still require a minimum concentration $\gamma_{min} = 5\%$ oxygen in every breath. We accept that successfully taking in 100% of the inhaled oxygen is likely impossible [1], but a detailed analysis of what concentrations of oxygen are required for survival would be beyond the scope of this paper, and so 5% is accepted as the bare minimum concentration required. As the snorkel is open to the atmosphere, the air within the snorkel and the lungs will be at atmospheric pressure, and so the value of 5% will remain the same.

Assuming a total lung volume $V_L = 8.6l$ [2] and also assuming in each stroke a human almost completely voids their lungs of gas, leaving a remaining $V_R = 29\% V_L = 2.5l$ [3], a human will take into their lungs a volume of $V_L - (V_S + V_R)$ of new air and $V_S + V_R$ of air from the previous stroke where V_S is the internal volume of the snorkel, assuming the air in the snorkel does not have sufficient time to mix with the air above the snorkel between strokes. The air from the previous stroke will be 21% oxygen to begin with, and 16% on the stroke after that, as 5% of the oxygen will have been removed. Therefore the oxygen concentration in any stroke n after the critical 5% of V_L has been removed will be

$$\gamma_n = \frac{(V_L - (V_S + V_R))\gamma_{air} + (V_S + V_R)\gamma_{n-1}}{V_L} - 0.05, \quad (1)$$

where γ_{n-1} is the concentration of oxygen in the previous stroke and γ_{air} the concentration of oxygen in the outside air. For large enough n, γ_{n-1} tends to γ_n and as such Eq. 1 tends to $\gamma_n = \gamma_{air} - \frac{0.05V_L}{V_L-(V_S+V_R)}$.

We can then fix $\gamma_n \geq \gamma_{min} = 0.05$ to give $V_S \leq \frac{11}{16}V_L - V_R = 3.4l$. This is the maximum volume of snorkel that will still allow enough oxygen in each stroke for survival. This can then be used to imply a maximum length given a certain snorkel diameter. Estimates of a snorkel diameter of about $2cm$ yields a maximum length L of $10.8m$, with larger bores implying shorter tubes.

Buoyancy

Next we consider the buoyancy force of the snorkel. Obviously a swimmer will not want to be constantly pulled to the surface of the water by their own breathing apparatus, and as such we shall find the length of snorkel at which the resulting upward force will be zero. We use an estimate of plastic density of roughly $1200kgm^{-3}$ [4], an estimate of water density of $1000kgm^{-3}$ [5], and an estimate of air density of $1.3kgm^{-3}$ [6]. The law of relative buoyancies then gives the total force to be

$$F = g(M_S + M_B - M_W), \tag{2}$$

where g is the acceleration due to gravity, $9.81ms^{-2}$, M_S is the mass of the snorkel and the air within, M_B is the mass of a human body, and M_W is the mass of water displaced by the snorkel and the human's body. The mass of a human body is taken to be $58.2kg$ [7] and its density to be $1070kgm^{-3}$ [8]. The buoyancy force of the snorkel can be written as $gM_S = g\mu L$ where μ is the mass per unit length of snorkel, $1.2kgm^{-1}$, found from the dimensions and densities described above. The force described in Eq. 2 must not be positive, else the diver would float, so we have the following relation,

$$L \leq \frac{M_W - M_B}{\mu}. \tag{3}$$

Eq. 3 then yields a maximum length of $L = 2.7m$. The human in question could obviously apply a downward force by swimming, enabling them to use a longer snorkel with greater buoyancy. We shall however consider this value of $2.7m$ as the upper limit, as this is the point at which the user would not be required to actively fight the buoyancy force.

Plastic Failure

Yield stresses for most engineering polycarbonates such as those used to construct snorkels are of the order $80MPa$ [9]. This is thirteen times greater than the pressure at which any human has ever survived diving to [10] [11], and as such we shall neglect this limit.

Conclusion

The smallest of these limits is $2.7m$ which is imposed by the snorkel buoyancy, although as discussed this is not an exact limit. There will also be limits imposed by others factors, such as the danger associated with accidentally introducing water into such a long snorkel. Furthermore to this, even a $2.7m$ snorkel would be incredibly difficult to manouevre with, and a SCUBA kit may be more appropriate. The limit imposed by the volume of the snorkel will be affected by the snorkel diameter. There will be a limit on the minimum diameter of tube through which air may be pushed by the pressure gradient generated by a human lung. This could be a topic of future investigation.

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