

P5_7 Put a Ring on it

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Abstract

This article investigates if a scaled replica of the rings around Saturn could be formed around the Earth and the Moon by considering moving Moon-rock or water ice into their Roche limits. It is found that a rigid satellite made from Moon-rock would not form the desired ring system around either the Earth or the Moon, though a liquid satellite of either Moon-rock or water ice would.

Introduction

It is very easy for inhabitants of Earth to be envious of the magnificent ring system of Saturn. Neglecting all possible downsides and practicalities, such as disruptions to sunlight and life on Earth or physical construction, this article investigates whether it is possible to have a replica of Saturn's main rings around the Earth or the Moon. The proposed method is moving satellites made of Moon-rock or water into the Roche limit where it will be broken apart by tidal forces into smaller particles to make the ring.

Theory

Tidal forces are secondary gravitational effects due to the force of gravity not being constant across a body. If the gradient across the body is large enough, the body gets torn apart by tidal forces.

The Roche limit gives the distance at which the tidal forces pulling an object from the surface of an orbiting satellite is exactly equal to the gravitational force holding it to the satellite. Within this distance objects on the surface of the satellite are gravitationally unbound. Hence, if a satellite is entirely within the Roche limit it will be broken apart by tidal forces and will form a ring around the primary mass that the satellite is orbiting. The Roche limit of a liquid satellite* is given by

$$d = 2.44 \cdot R_M \left(\frac{\rho_M}{\rho_m} \right)^{\frac{1}{3}}, \quad (1)[1]$$

where d is the distance from the surface of the satellite to the centre of mass of the primary mass, R_M is the radius of the primary mass, ρ_M is the density of the primary mass and ρ_m is the density of the satellite.

A rigid satellite* would require stronger tidal forces to pull it apart, and hence the Roche limit for a rigid satellite is less than that for a liquid satellite and is given by

$$d = 1.26 \cdot R_M \left(\frac{\rho_M}{\rho_m} \right)^{\frac{1}{3}}. \quad (2)[1]$$

Ring Properties

The main rings of Saturn are the D – A rings (ordered alphabetically from their date of discovery) and are made primarily of water ice [2]. The radial dimensions of replica rings around the Earth are estimated by scaling the proportions of the D – A rings.

	Saturn (km)	Earth (km)
Radius	58,200 [3]	6,370 [4]
D, inner edge	67,000 [5]	7300
A, outer edge	137,000 [5]	15,000
Total width	70,000	7700

Table (1); Ring dimensions for the Earth.

Keeping the average thickness of the main rings, 10m [2], the same for rings around Earth, the volume of space occupied by the rings is $5.4 \times 10^6 \text{ km}^3$. It is important to note that rings are not solid bodies, and are made of many individual chunks of rock and ice. [2]

*A liquid satellite is a satellite that is bound together by gravity only, whereas a rigid satellite has tensile strength also. Note that a liquid satellite is not necessarily in the liquid state of matter.

Moon Rock

A relatively small amount of the Moon's volume of $22.0 \times 10^9 \text{ km}^3$ [6] could be used to form the rings. The percentage needed is less than 0.025% of the Moon to make even a solid ring body, which does not occur in nature, so it is assumed that the changes to the tidal effects of the Moon's gravity on Earth and its oceans would be negligible.

For a solid body of Moon-rock to break apart, it must lie within the rigid satellite Roche limit. To calculate the Roche limit for an Earth/Moon-rock system, we need their densities for equations (1) and (2). The density of the Earth is $5,513 \text{ kgm}^{-3}$ [4] and the density of the Moon is $3,344 \text{ kgm}^{-3}$ [6]. This gives a Roche limit for a rigid satellite of 9,500km. Whilst a portion of the ring system lies within this limit, a good deal does not. This means that the desired replica of Saturn's ring system made from Moon-rock around the Earth is not possible by using a solid body of Moon-rock.

However, the Roche limit for a liquid satellite is 18,400km. Since the outer edge of the replica ring system is within this limit, if the Moon-rock was only gravitationally bound together, the desired ring system could form.

Water Ice

Assuming the water would freeze in space, if water was removed from the sea to create the rings, the Roche limit should be larger than the Earth/Moon-rock system, as the density of water ice is 917 kgm^{-3} [7], neglecting salinity.

The Roche limit for a liquid satellite in this system is 28,300km, meaning a replica ring system is possible as the whole of the ring system lies within the Roche limit.

A Ring around the Moon

In this section, creating a replica ring system around the Moon is considered. Again, the dimensions of Saturn and its ring system are scaled to that of the Moon.

	Saturn (km)	Moon (km)
Radius	58,200 [3]	1740 [6]
D, inner edge	67,000 [5]	2000
A, outer edge	137,000 [5]	4100
Total width	70,000	2100

Table (2); Ring dimensions for the Moon.

The following Roche limits are calculated based on the Moon's radius and density:

- Rigid satellite of Moon-rock: 2200km
- Liquid satellite of Moon-rock: 4200km
- Liquid satellite ice water: 6500km

From this it is determined a Saturn ring system replica around the Moon is not possible for a rigid satellite made of Moon-rock, as for Earth. However, liquid satellites of both Moon-rock and water ice would be able to form rings.

Conclusion

By analysing scaled ring sizes and the Roche limits of several systems, it is concluded that replicas of Saturn's rings can be formed around the Earth and Moon if the bodies forming them are solely bound by gravity. Solid satellites, would not form the desired ring systems. It should be noted, however, that the gravitational effects of the Earth/Moon system on the ring systems of either body are not investigated in this article.

References

- [1] F. H. Shu, *The Physical Universe: An Introduction to Astronomy* (University science books 1982), p.431.
- [2] http://www.ciclops.org/sci/common_questions.php?js=1#ring accessed on 23/10/2013.
- [3] <http://solarsystem.nasa.gov/planets/profile.cfm?Object=Saturn&Display=Facts&System=Metric> accessed on 23/10/2013.
- [4] <http://solarsystem.nasa.gov/planets/profile.cfm?Object=Earth&Display=Facts&System=Metric> accessed on 23/10/2013.
- [5] <http://www.universetoday.com/15300/wh-are-saturns-rings-made-of/> accessed on 23/10/2013.
- [6] <https://solarsystem.nasa.gov/planets/profile.cfm?Object=Moon&Display=Facts&System=Metric> accessed on 23/10/2013.
- [7] <http://www.convert-me.com/en/convert/density/densice.html> accessed on 23/10/2013.