Journal of Physics Special Topics

P5_4 Extreme Altitude Wind Power

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October 11, 2013

Abstract

This article investigates the use of the lift created by rigid aerofoils in altitudes of 1km to 7km as a method of generating energy. From the assumptions made by atmospheric and aerofoil models described in the article, an aluminium aerofoil is only effective up to a thickness of 220 µm. This is considered too small to create feasible rigid foils.

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Introduction to High Altitude Wind Power

Makani Power is a company that produces electric power using airborne turbines*[1] at higher altitudes than conventional turbines. The reason given for this is that at higher altitudes wind speeds are greater and more consistent. The Makani approach uses airborne turbines at altitudes between 120 - 360m. [1] In this paper, we investigate the possibility of using a simple aerofoil design at higher altitudes.

Lifting Against Gravity

To find out if energy can be drawn from these greater altitudes, the work done by a rising aerofoil is compared to the work that must be done against gravity. Assuming the change in acceleration due to gravity between altitudes considered is negligible, these are given by equations (1) and (2) respectively:

$$W_{lift} = \int_{h_1}^{h_2} F_{lift}\, dh, \qquad (1)$$

$$W_{grav} = -mg\Delta h, \qquad (2)$$

where h is the altitude, h_1 and h_2 are the lower and upper limits of the altitude respectively, W_{lift} is the work done by a rising aerofoil, F_{lift} is the force of lift, W_{grav} is the work done against gravity, m is the mass of the aerofoil and g is the acceleration due to gravity (9.81 m/s^2).

F_{lift} is given by:

$$F_{lift} = \frac{1}{2}\rho_{air} v^2 A C_{lift}, \qquad (3)$$

where ρ_{air} is the density of air, v is the velocity of the aerofoil in relation to the medium, in this case it is the wind speed, A is the area of the aerofoil generating lift and C_{lift} is the coefficient of lift.

Altitude Variations

However, the wind speed and the density of air are both functions of altitude, though it is assumed that neither is a function of time. The density of air as a function of altitude is given by the change of pressure and temperature with altitude, equations (4) and (5) respectively. Note that equation (5) is only valid for the troposphere, which has a maximum altitude of 7km in the winter at the poles and 20km at the tropics.

$$p = p_0 \left(1 - \frac{Lh}{T_0}\right)^{\frac{gM}{RL}}, \qquad (4)\ [2]$$

$$T = T_0 - Lh, \qquad (5)\ [2]$$

where p is the pressure at altitude h, p_0 is the sea level pressure, (101.325kPA), T_0 is the sea level temperature (288.15K), L is the temperature lapse rate (0.0065K.m^{-1}), R is the ideal gas constant (8.314 J.mol^{-1}.K^{-1}), M is the molar mass of dry air (0.02896 kg.mol^{-1}), and T is the temperature at altitude h. Density is then given by:

$$\rho_{air} = \frac{pM}{RT}. \qquad (6)\ [2]$$

**[1]*An airborne turbine is a rigid aerofoil that rotates around an axis, similarly to the tip of a regular turbine, pulling on a tether to generate power. [1]*

Substituting equations (4) and (5) into (6) will give a relation for density of air with altitude, which will later be applied to equation (3).

Now a relationship for wind speed and altitude must be found. Data from the national weather service for Riverton, Wyoming [3] is evaluated and an approximate linear relationship is taken for altitudes between 1km and 7km. Taking wind speeds of 8m/s at 1km and 18m/s at 7km results in equation (7). An altitude of 7km is taken as maximum as after this the tropospheric model for atmospheric temperature begins to fail.

$$v = 0.00167h + 6.33. \quad (7)$$

Now, equations (4) and (5) are substituted into (6), then (6) and (7) are substituted into (3), resulting in equation (8):

$$F_{lift} = \frac{1}{2}(0.00167h + 6.33)^2 \frac{p_0\left(1 - \frac{L}{T_0}h\right)^{\frac{gM}{RL}}}{R(T_0 - Lh)} AC_{lift}. \quad (8)$$

This equation can be integrated as per equation (1) to obtain the work done by the aerofoil.

Equation (8) is substituted into equation (1) and integrated between the altitudes of 1km and 7km, resulting in the approximate relation that:

$$W_{lift} = 16{,}100 \cdot AC_{lift}. \quad (9)\ [4]$$

In order to get the useful energy, the work done against gravity must be taken into account, giving:

$$W_{net} = 16{,}100 \cdot AC_{lift} - 58{,}900 \cdot m. \quad (10)$$

Aerofoil Model

Now a model must be created for the aerofoil. The Clark Y-14 foil design is described as "[a] general purpose aerofoil chosen for superb control at low Reynolds numbers" [5]. This foil has a maximum coefficient of lift of approximately 1.1 given the correct angle of attack. [6] For mass and area calculations, The Clark Y-14 is approximated as having a cross-section given by figure (1).

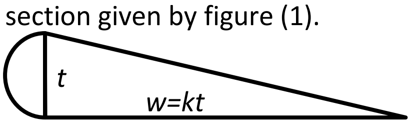

Figure (1); foil cross section model.

The span of the foil is given by *s*, and hence the area of the foil generating lift is given by $A = ws$. For the geometry of the foil, w is said to be equal to *k* times larger than *t*. For sufficiently large *k*, the semi-circular edge can be neglected when calculating the volume.

Applying this model to equation (10) gives:

$$\frac{W_{net}}{ks} = 17{,}700t - 29{,}450t^2\rho_{foil}, \quad (11)$$

where ρ_{foil} is the density of the material that the foil is constructed from. In this equation, ks is just a multiplier brought about by scaling the area of the foil. The values that affect whether or not the foil will generate energy are the thickness of the wing and its density.

For the maximum power to be generated, the foil must have a thickness given by equation (11) where ρ_{foil} is in kg/m^3.

$$t = \frac{0.3}{\rho_{foil}}\ metres, \quad (12)$$

Additionally, the foil only gives positive net work up to a thickness given by:

$$t = \frac{0.6}{\rho_{foil}}\ metres. \quad (13)$$

Results

To calculate a value for t, the density of aluminium is used as it is a lightweight, sturdy material. In equation (13) this gives *t=110μm*. This is very small. Additionally an aluminium foil also gives no net energy after a thickness of *t=220 μm*. Denser materials would yield even smaller thicknesses. From this model it is concluded that rigid aerofoils are not the best choice to generate power from extreme altitude winds.

References

[1] http://www.makanipower.com/home/ accessed on 16/10/2013.
[2] http://wahiduddin.net/calc/density_altitude.htm#b14 accessed on 16/10/2013.
[3] http://www.classzone.com/books/earth_science/terc/content/investigations/es1702/es1702page09.cfm accessed on 12/10/2013.
[4] http://www.wolframalpha.com/ accessed on 16/10/2013.
[5] G. Bruschi et. al. *Clark Y-14 Airfoil Analysis* (2003) section (1.4) p. 8.
[6] G. Bruschi et. al. *Clark Y-14 Airfoil Analysis* (2003) figure (10), p. 39.