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P1_1 "Everybody knows the moon is made of cheese..."

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Abstract

This report investigates the possible implications of our moon being made of cheese as suggested in the Wallace and Gromit film "A Grand Day out". If it were the same size as the current moon, and made of Wensleydale cheese, then it would exert 13.1 $\times 10^{19}$N less force on Earth. If it were to exert the same force then its radius would increase by 0.78 x 10^{6}m, appearing 144% larger in the night sky.

Introduction

In the 1989 animated film Wallace and Gromit's "A Grand Day Out" the pair discover that they have completely run out of cheese. Their solution to this problem is to take a trip to the moon. When they arrive on the moon Wallace announces that the moon-cheese tastes very similar to Wensleydale cheese. If the moon was truly made of Wensleydale cheese what effects would this have on the Earth? Through the application of different constraints, the change in the force of gravity and the moon's appearance in the night sky can be calculated for that of Wensleydale cheese and other cheeses.

Discussion

If our moon was instantaneously turned into cheese this would change the density, thus impacting the volume and mass of the moon. We have investigated these changes holding one variable constant each time.

The relationship between these three variables is as follows;

$$m = \rho V \, , \qquad (1)$$

where m is the mass, ρ is the density and V is the volume.

The densities of the cheeses investigated were calculated using a rearrangement of equation (1). By measuring the dimensions and weighing a small sample of each cheese, the volume and mass can be calculated. The resulting densities have been recorded in tables 1 and 2.

Constant Volume

In the case where the volume of the moon is kept constant the mass will be forced to change with the changing density. The volume of a sphere is defined as

$$V = \frac{4}{3}\pi r^3, \qquad (2)$$

where V is the volume and r is the radius. By substituting equation (1) into equation (2) and rearranging for the mass we get

$$m = \frac{4}{3}\rho\pi r^3. \qquad (3)$$

We know from Newton's law of universal gravitation that

$$F_g = \frac{GmM}{r^2}\, . \qquad (4)$$

Where F_g is the force of gravity, G is the gravitational constant ($6.67\times10^{-11}m^3kg^{-1}s^{-2}$), m is the mass of the moon, M is the mass of the Earth (5.972×10^{24}kg) and r is the distance between the two masses (3.84×10^{8}m). By substituting equation (3) into equation (4) we get the following;

$$F_g = \frac{GM}{r^2}\left[\frac{4}{3}\rho\pi r^3\right]. \qquad (5)$$

Constant mass

In this case we will keep the mass of the moon constant and see how this will change the radius and therefore how it would look to an observer on Earth.

By rearranging Equation (1) and putting it equal to equation (2) we get

$$V = \frac{m}{\rho} = \frac{4}{3}\pi r^3\, , \qquad (6)$$

which can be further rearranged to give the radius

$$r = \sqrt[3]{\frac{3\ m}{4\ \pi\rho}}\ . \qquad (7)$$

Assuming that the orbital radius of the moon stays the same, the appearance of the moon in the night sky will change with the changing radius. In order to calculate this change we used the angular diameter formula

$$\delta = 2\ \tan^{-1}\frac{d}{2D}\ , \qquad (8)$$

where δ is the angular diameter or apparent size, *d* is the true diameter of the moon, *D* is the distance between the centre of the moon and centre of Earth. This result was then converted into arcminutes by

$$\delta_{arcmin} = \delta_{radian} \times 60 \times \frac{180}{\pi}. \qquad (9)$$

Results

Constant Volume	Wensleydale	Cheddar	Swiss	Current Moon
Density (kg/m^3)	1111.11	1035.55	917.21	3346.40
Lunar Mass (kg x 10^{22})	2.44	2.27	2.02	7.34
Force exerted (N x 10^{19})	6.60	6.12	5.44	19.67

Table 1 – The results from using equations (1), (3) and (5).

Constant Mass	Wensleydale	Cheddar	Swiss	Current Moon
Density (kg/m^3)	1111.11	1035.55	917.21	3346.40
Volume (m^3 x 10^{19})	6.61	7.05	8.01	2.199
Radius (m x 10^6)	2.51	2.56	2.67	1.73
Angular diameter (arcmin)	44.93	45.96	46.58	31.00

Table 2 – The results from using equations (1), (7), (8) and (9).

Analysis

When the volume was kept constant the mass of the Wensleydale moon was 4.9x10^{22}kg (66.8%) less than that of the current moon. This led to the force exerted on the Earth by the moon being decreased by 13.1 x10^{19}N. This drastic change would completely change the tidal forces on Earth.

When the mass was kept constant, the radius of the Wensleydale moon would be increased by 0.78 x 10^6m; 145% the size of the current moon. If this increase did occur, the apparent size of a full moon in the night sky would be approximately 144% larger than what is normally seen.

Conclusion

If the moon were truly made out of Wensleydale cheese then either our tides would be drastically different or the moon would be a lot bigger. The other cheeses had similar densities to Wensleydale and therefore show similar results. Although both Cheddar and Swiss are marginally further from the current Moon's characteristics than Wensleydale.

References

[1] http://wallaceandgromit.wikia.com/wiki/A_Grand_Day_Out accessed on 08/10/2013

[2] http://en.wikipedia.org/wiki/Angular_diameter#Formula accessed on 08/10/2013