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A2_8 Big M.A.C.

M. Bryan, J. Forster and A. Stone

Department of Physics and Astronomy, University of Leicester. Leicester, LE1 7RH.

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Abstract

This paper attempts to model the amount of energy transmitted to air by drag when a Magnetic Accelerator Cannon, as described by the Halo series of games and books, is fired vertically in atmosphere. For a normal MAC gun, the air is heated by 96,000K, enough to cause a column of plasma to form, expanding at Mach 22 in the form of a shockwave. The energy of impact is also found to be 60 kilotons, similar to a low yield nuclear blast. Comparatively the Super MAC is found to create near relativistic shockwaves, with enough energy to cause combustion of the atmosphere ($E_D \approx 1.051 \times 10^{18} J$). The impact would be equivalent to 51 Giga-tons of TNT.

Introduction

In the Halo series of games, the weapon known as a Magnetic Accelerator Cannon (MAC gun) is frequently featured, being the weapon of choice for the humans in ship to ship combat. These are simple Gauss guns, but on a large scale, utilising super-conducting coils to accelerate tungsten or depleted uranium charges to vast speeds in order to punch through enemy ship's armour and shields. These vary in size from the mini-MAC mounted on ground assault vehicles, to the Super MAC housed in orbital defence platforms, designed to destroy capital ships in a single shot. However, in the game Halo: Reach we see a cruise mounted MAC gun fired from orbit into atmosphere, destroying a frigate and producing a fairly wide, but also fairly undamaging shockwave. This paper will attempt to model the actual effects of such a strike for both the standard cruiser MAC gun, and the substantially more powerful Super MAC when fired into the atmosphere of the planet Reach, which will be modelled as Earth for simplicity.

Determining the drag

A standard MAC gun is quoted as capable of accelerating a 9.1m long, 600-Ton, tungsten shell to $30 Kms^{-1}$ [1], giving it an initial kinetic energy of $E_k = 2.7 \times 10^{14} J$. Assuming it is firing from a low Earth orbit of 2000km into Earths atmosphere, bearing in mind that the Earth's gravitational energy will be noticeably lower at this point, the shell has an initial potential energy of $E_u = 6.824 \times 10^{12}$ J. By taking the approximate start of the atmosphere to be at 100km, we can determine the velocity as the shell enters the atmosphere by the change in potential that it experiences:

$$v = \left(\frac{2}{m} \left(E_k + E_u - \frac{GM_E m(10^5)}{(R_E + 10^5)^2} \right) \right)^{\frac{1}{2}}, \qquad (1)$$

where M_E is the mass of the Earth and R_E is the Earth's radius. This results is a new velocity $v = 30.35 kms^{-1}$, which is not a particularly large difference, meaning we can assume that within the atmosphere, the velocity of the shell should not change significantly and can be assumed constant (note that past this point the gravitational acceleration can be assumed to be $9.81 ms^{-1}$). When calculating the drag on the shell, it's important to note that the density of the atmosphere is dependent on the height at which the shell is located [2]:

$$F_D = \frac{1}{2} C_D \rho v^2 A = \frac{1}{2} C_D v^2 A e^{-x/x_0} \frac{MP_0}{RT}, \qquad (2)$$

where C_D is the drag coefficient, which in this case will be 0.04 for a streamlined conical object [3]; A is the surface area facing the atmosphere, which can be determined by modelling the shell as a 9.1m long cone of tungsten at density $19000 kgm^{-3}$, giving a conical face of $52.48 m^2$; M is the average molar mass of air ($= 0.0289 kgmol^{-1}$), R is the molar gas constant, P_0 is the sea-level air pressure ($= 101325 Pa$), T is the average atmospheric temperature, as the temperature gradient of the Earth's atmosphere has several discontinuities at the different layers and so needs to be averaged over all layers ($\approx 239.25 K$) ; x is the height of the object and finally x_0 is the scale height, which for the Earth is approximately 7km. To find the energy that the drag imparts to the air during descent, we simply integrate the drag force over the 100km altitude:

$$E_D = \frac{1}{2} C_D v^2 A \frac{MP_0}{RT} \int_0^{10^5} (e^{-x/x_0} dx), \qquad (3)$$

which gives $E_D \approx 1.016 \times 10^{13} J$.

Calculating the effects

By modelling this energy as being transferred to the air displaced by the column of radius r (the radius of the conical shell $r = 1.8m$) swept out during the shell's descent we can determine the temperature change, using a specific heat capacity of $C = 1003.5Jkg^{-1}K^{-1}$ [4] and hence the thermal speed of the air outwards from the column:

$$T = E_D \left(C\pi r^2 \frac{MP_0}{RT} \left[-x_0 e^{-x/x_0} \right]_0^{10^5} \right)^{-1}, \quad (4)$$

calculated to be 96422K, which is then put into the equation:

$$v_A = \sqrt{\frac{2k_B \Delta T}{m}}, \quad (5)$$

where m is the average mass of a single air molecule, k_B is the Boltzmann constant and v_A is the approximate thermal velocity of the air out wards from the column, calculated to be $7391.8ms^{-1}$ or $\sim$ Mach 22.7. The first thing that can be surmised by this temperature change is that the air will be turned to plasma. This can be determined quite easily as a lightning strike on average heats the air it passes through by about 30,000K [4], which is observed to cause thermal decomposition of the air into ions and electrons, forming a plasma. By logical extension a temperature increase of 96,422K will easily decompose the air into plasma. From the thermal speed imparted to the air we can see that the column of air will expand at a supersonic rate. The pressure of the resulting shockwave can be determined fairly easily using standard shockwave equations [5]:

$$p_s = p_0 \frac{2\gamma M_N^2 - (\gamma - 1)}{\gamma + 1}, \quad (6)$$

where p_0 is the atmospheric pressure ($= 101325Pa$), M_N is the mach number and γ is the ratio of specific heats, which for high temperature air is approximately 1.3. This gives a p_s of $\approx 59MPa$. Since we know how much energy is lost to drag, we can determine the final kinetic energy with which the shell strikes the ground:

$$E = E_k - E_D \approx 2.5 \times 10^{14} J. \quad (7)$$

To put this value into perspective we can convert into TNT mass equivalent units (i.e. 1 ton TNT$= 4.184 \times 10^9$J), which gives us a blast force equivalent to 59.7 kilotons of TNT, approximately four times the power of the 15 Kiloton Little Boy bomb dropped on Hiroshima.

Discussion

In the game Halo: Reach, the camera is located in a light aircraft hovering fairly close to the location of the MAC gun strike; assuming that the shockwave does not slow down by a significant amount in the 1km (approximately) distance from the strike point to the aircraft, and modelling the aircraft with a surface area of about $2m^2$ facing the incoming shockwave, we get an impacting force from the air of $\sim 118MN$. However, the craft is not even shaken by the blast, which, given the known TNT equivalence, is visibly far too small to be a 59.7 Kiloton blast. Clearly either an oversight by the game developers, or artistic license taken to account for the fact that, with these calculated values, within a few miles of the impact zone, the game's players would be killed near instantly. However, this is not the worst situation that could occur. Elsewhere in the series we are introduced to the so-called Super MAC gun, which fires a 3000 ton shell from an orbital defence platform in geosynchronous orbit of 35000km at 0.04 times the speed of light. Using the same method used for the standard MAC gun, assuming the shell has similar proportions just scaled for the new weight, it can be determined that the round would cause a drag energy $E_D = 1.051 \times 10^{18}$J and a thermal expansion speed of ~ 0.02 times the speed of light. The air around the path of descent would be near instantaneously ionized, and due to the sheer level of energy imparted to the air, nitrogen, oxygen and several other molecules may undergo thermolysis; a process whereby molecules under high heat decompose into single atoms, which would allow for atmospheric combustion, a process that would burn everything within a several mile wide radius. This level of energy could also not be purely modelled as a temperature increase, as the air would experience $\Delta T \approx 10^{10}$K, a temperature rise that would be far too large for any real system. This pales in significance to the energy of the shell's impact, $E \approx 2.16 \times 10^{20}$J, equivalent to 51.6 Giga-tons of TNT [6]. This is approximately the energy released in an as yet unobserved Richter-scale magnitude-10 earthquake, which involves the surface rupturing along an entire major fault.

Conclusion

In conclusion, while a standard MAC gun can be used as an easy alternative to a nuclear weapon by firing upon a planet, the Super MAC would cause far too much damage to be tactically viable, creating global levels of devastation. We can also conclude that the representation of a M.A.C. gun within the Halo games is far less powerful than suggested by their provided specifications

References

[1] Eric Nylund, *Halo: The Fall of Reach*, DelRey books.

[2] `http://acmg.seas.harvard.edu/people/faculty/djj/book/bookchap2.html` accessed on 21/11/2012

[3] `http://www.engineeringtoolbox.com/drag-coefficient-d_627.html`, accessed on 21/11/2012.

[4] D. Lide, Handbook of Chemistry and Physics 1996, crc press inc.

[5] `http://www.grc.nasa.gov/WWW/BGH/normal.html`, accessed on 21/11/2012

[6] `http://crack.seismo.unr.edu/ftp/pub/louie/class/100/magnitude.html` accessed on 21/11/2012