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A2_7 The falling Iron Sky

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Abstract
In the movie "Iron Sky", during the finale, the Nazi superweapon "Götterdämmerung" fires its main weaponry at the horizon of the Moon, blasting away a portion of the lunar surface and revealing the Earth above the new horizon. This paper calculates that the energy required for such a feat- on order of $10^{23}J$, would mean it would pose a critical threat to Earth.

Introduction
During the final act of the satirical film "Iron Sky", a huge, city-sized flying fortress is revealed- the Nazi superweapon Götterdämmerung. It shows startling resilience- being hit by several low-yield nuclear devices without taking any visible external damage (although several miles away, the control room lights are damaged). Its party piece, however, has to be its main guns, of which there are two mounted at the front of the vessel. These weapons are fired only once in the movie, but their effects are cataclysmic. When used to destroy part of the Moon to allow a clear shot at the Earth, they tear an enormous hole in the lunar surface, sufficient to view the Earth.

Impulse calculation
Given that we know that the Earth is just below the horizon, we can work out the volume of the moon that was blown away in the explosion. Geometry gives us the equation for horizon distance, d, for a given height to be

$$d = \sqrt{h(D+h)} \tag{1}$$

where h is the height of the observer from the surface and D is the diameter of the sphere, in this case we use the lunar diameter 3476km [1] and we say that the main cannon of the Götterdämmerung is 1km above the surface of the moon, giving us a horizon distance of 59km. At lunar distance the Earth has an angular size of 0.0375 radians (from its diameter) so assuming the Earth was just touching the horizon, the blast would have to remove a spherical cap 2km deep (h) in order to see the Earth. The equation for the volume of a spherical cap is given by

$$V_{cap} = \frac{1}{3}\pi h^2(3R - h) \tag{2}$$

which gives a volume of $2.2 \times 10^{13}m^3$. Given that the average density of the moon is $3.3 \times 10^3 kgm^{-3}$ [1] this gives a total mass ejection of $7.5 \times 10^{16}kg$. Assuming this mass is travelling fast enough to escape the moon's gravitational pull, this would make its velocity at least 2.4km/s [1], meaning that a lower estimate the total momentum of the ejected mass is $1.8 \times 10^{20}Ns$.

Specific impulse of the weapon
In order for this momentum change the explosive would have to have an enormous specific impulse. No specific scale information exists for the Götterdämmerung but visual estimation place the size of the projectiles to be around 100m in diameter. A scientist in the movie is overheard saying that the projectiles are metallic hydrogen, which has a density of around $1 \times 10^3 kgm^{-3}$ [2] meaning each of the two projectiles fired would have a mass of around $5 \times 10^8 kg$. The theoretical specific impulse for the atomisation of metallic hydrogen is only around $1000Ns/kg$ [2], nowhere near enough to provide the energy required to blast apart the moon. A rough estimate of the energy required can be estimated from the momentum and mass of the ejected material to be on order of $10^{23}J$ (assuming that the binding energy of the rock is negligible at that scale), well outside even the most powerful warheads ever produced on Earth, even if the metallic hydrogen could be forced to undergo fusion by some unknown mechanism, the energy yield would be (for complete protium-protium fusion to deuterium) on order of $10^{17}J$ still far from enough energy to remove that amount of Moon. Even a more complex fusion chain would still fail to produce the energy required with the mass of hydrogen estimated. The creators of the movie descibe

the weaponry as "relativistic" which would infer that a large portion of the energy in fact comes from the kinetic energy of the projectile.

Discussion

Although initially in the movie it appears that far less than a total spherical cap is removed from the Moon, during the credits we see that considerably more of the moon was destroyed than estimated (which can be seen in figure 1). Such destruction should not be physically possible with even thermonuclear weapons (the largest of which, Tsar Bomba, had an approximate energy of 10^{18}J [3]), as such it could be theorised that the explosion triggered the fusion of the Helium-3 in the Moon so prevalent to the plot. If this were to happen in a large enough pocket, the resulting reaction should produce enough energy. Additionally large pieces of the Moon appear to be orbiting it in the same scene, which would mean that these chunks of lunar rock had not escaped from the lunar gravity well. With such a large outflow of matter, it is quite possible that the orbit of the Moon would be changed considerably, with quite a lot of the mass heading inwards of the orbit, it would be sufficient to increase the rate at which the Moon's orbit is degrading outwards by a considerable amount (around 6cm/s). The debris from the fallout of the Moon could have serious impact on Earth if any large pieces of lunar material were accelerated sufficiently to escape the Moon's gravitational pull in the direction of Earth. Although such considerations are moot as later in the end credits we see the world in the process of thermonuclear war, making such impact events a minor concern.

Fig. 1: The damage done to the moon at the end of Iron Sky

Conclusion

The energy required to devastate the moon would have to primarily come from the kinetic energy of the projectiles rather than any chemical or nuclear source. The effects of the damage done to the moon could have serious consequences for Earth both from the debris and the eventual loss of the satellite itself. With such displayed power, the Götterdämmerung would have posed a serious threat to Earth had it made it more than a few minutes into its flight. Although what use as a weapon of conquest it would have been is questionable as if its effect on the Moon is to be considered representative of its power, the Götterdämmerung would have caused apocalyptic amounts of damage to the Earth itself, making an invasion rather difficult.

References

[1] `http://solarsystem.nasa.gov/planets/profile.cfm?Display=Facts&Object=Moon` accessed on 21/11/2012.

[2] `http://www.nasa.gov/pdf/637123main_Silvera_Presentation.pdf` accessed on 21/11/2012.

[3] V. Khalturin, T. Rautian, P. Richards, W. Leith, *A Review of Nuclear Testing by the Soviet Union at Novaya Zemlya, 1955-1990*, Science and Global Security, 13:142, (2005).