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A1_3 The Penny's Dropped

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Abstract
This paper investigates the common myth of whether dropping a penny from the top of the empire state building could kill a person that it hits at the bottom. The paper deals with a standard sterling penny dropped from the 102nd floor of the empire state building at 381m above ground level. The penny would not be able to reach a sufficient speed to kill or badly injure anybody walking below but would certainly be painful.

A1 The Penny's Dropped

Introduction
Due to the extraordinary height of the empire state building located in midtown Manhattan, New York City, it is assumed that an ordinary object such as a penny, when dropped, could reach high enough speeds to become dangerous to innocent bystanders at the foot of the tower, potentially even killing them. To investigate this we need to take into consideration the height of the tower and some basic properties of a standard sterling penny including its terminal velocity and the air resistance acting upon it. We can therefore determine its force on impact.

Theory
When an object is dropped from any height, as long as it is released at t=0 and with initial velocity v=0, then its velocity at time t can be calculated using

$$v(t) = \sqrt{\frac{2mg}{\rho A C_d}} tanh\left(t\sqrt{\frac{g\rho C_d A}{2m}}\right). \quad (1)$$

where *m* is the mass of the object, *g* is the gravitational constant (9.81), ρ is the density of the medium the falling object is travelling through and C_d is the drag coefficient of the object being dropped.
This function involves the hyperbolic tangent *tanh*. For large values of t the value of the *tanh* tends to 1, this is its limit. Physically what is happening is the object has reached its terminal velocity and therefore no matter the length of *t*, v cannot increase any further. For this reason the above equation can be simplified to

$$v(t) = \sqrt{\frac{2mg}{\rho A C_d}}. , \quad (2)$$

where the symbols mean the same as in Equation (1). Finally, to calculate the energy the object has when it hits, the equation used is

$$E = \frac{1}{2}mv^2. \quad (3)$$

Discussion

The highest floor easily accessible in the empire state building is the observatory deck on the 102nd floor. This stands at a height of 381m above the pavement [1]. We can calculate the pennies terminal velocity using equation (2) for the calculations. The standard weight of a sterling penny is 0.00356kg [2] and the area of the face upon which air resistance is acting is $3.08X10^{-5}$ m2 [3]. Assuming the penny tends the case where the flat plate is parallel to the flow then the drag coefficient for a small cylinder is given as 0.115 [4], and the density of air is $1.225kgm^{-3}$ at sea level and at $15^{o}C$. The flow can be said to be laminar because we are assuming constant fluid motion of the air and neglecting the substantial impact of eddies. Using Equation (2) the terminal velocity of the penny with which will hit the ground (or person walking below) is

$$v(t) = 40.121ms^{-1}$$
$$\text{approximatly } 90mph.$$

Therefore the energy of the penny, found by inserting the result for v(t) into equation (3) is

$$E = 2.86J \text{ or } 804J/kg$$

Conclusion

The results show that the penny has a Kinetic Energy of 2.86 Joules on impact which can be translated into a specific energy of 804 joules per kilogram. This is not enough to kill someone or even badly injure them/ Since it reaches terminal velocity in this case, no matter what height the penny is dropped from it will not kill anyone.

References

[1] http://www.esbnyc.com/explore_esb_about_esb.asp accessed on October 15th 2012.

[2] http://www.royalmint.com/discover/uk-coins/coin-design-and-specifications/one-penny-coin accessed on October 15th 2012.

[3] http://www.royalmint.com/discover/uk-coins/coin-design-and-specifications/one-penny-coin accessed on October 15th 2012. Found the area from the diameter multiplied by the thickness of the newest pennies in circulation.

[4] http://www.engineeringtoolbox.com/drag-coefficient-d_627.html accessed on November 3rd 2012.