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A3_1: How long for a ship to sink?

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Abstract
Certain ships take longer to sink than others due to differing masses and other such factors [1]. The determination of the time taken to sink a ship/boat is not straightforward, so we make some simplifying assumptions and use an algorithm to estimate this time taken.

Introduction
The sinking of ships is a fascination to a great many people; an obvious example being appetite people still have for the spectacle of the sinking titanic in 1912 [2]. This paper will formulate a model to predict the time taken for a generic ship to sink and then apply it to a Spanish galleon which has gained some interest over the years [3].

Theoretical assumptions asserted to formulate the algorithm
To simplify the problem we assume that:

The ship is floating (and sinking) in fresh pure water with a viscosity of zero and constant density.

The ship is perforated at its lowest point in the water and the perforation is circular and symmetrical about this point.

The area of the perforation in the ship's hull is constant from the moment of perforation until the moment of complete submersion.

Water flows vertically upwards relative to the surrounding body of water containing the ship, in the reference frame of the ship; and of course the ship sinks in the sense of vertically downwards in the reference frame of the surrounding water body.

The Bernoulli equation is valid for the water flow throughout the ship.

The time taken to sink can be found by dividing the ship into N horizontal segments, small enough such that the acceleration of water flow in the vertical direction through a segment can be considered slow enough to be negligible. More segments means a more accurate time calculated for the ship to sink.

Also, to simplify the ships geometry we assume that the ship is of hemispherical shape of radius, r_S, floating in the water on its curved face, whilst its flat face is the top deck.

Algorithm used to calculate the time taken to sink the ship
The initial density of a ship of given mass and volume (and hence, radius r_s) can be calculated using [4]:

$$m_{S0} = \rho_{S0}V_{S0} = \rho_{S0}\left(\frac{2\pi r_S{}^3}{3}\right). (1)$$

Also, the initial volume of displaced water V_{Wo} can be found from:
$$m_{S0} = \rho_{W0}V_{W0}, (2)$$
where the subscripts S and W refer respectively to the ship and the water displaced by the ship. Also note that in the algorithm the 0 subscript means the initial (mass, volume etc.) whereas the subscript i means the situation at the ith subsequent segment (measured from the bottom of the ship upwards) that has been filled by water. Spherical geometry then gives the depth at which the ship rests at in the water, H_i [5]:

$$\Delta m_s = \rho_W V_W = \rho_W\left(\frac{2\pi H_i{}^3}{3}\right)(3r_S - H_i), (3)$$

using $V_W = V_{Wo}$, for the first segment to find H_0, the initial depth etc. Then, make use of the Bernoulli equation [6]:

$$\frac{v_{ei}}{2} + gz_{ei} + \frac{P_e}{\rho_e} = \frac{v_{ai}}{2} + gz_{ai} + \frac{P_a}{\rho_a}, (4)$$

the quantities v, g, z, P, ρ, A, respectively represent velocity of water flow (in vertical direction), acceleration due to gravity near the Earth's surface, vertical distance of descent of the ship into the water (after perforation), pressure, density, and area of entrance or exit to a given ith segment. The subscripts e and a respectively represent the position of entrance of water into a segment, and position of exit of water from a segment. z_{ai} is definitively equal to zero. Also, note z_{ei} is a positive measure of how far the ship sinks below the water line after the ith segment is filled. One can know both v_{ei} and A_{ei} (possibly from measurement) at the perforation site, and after calculate A_{ei} and A_{ai} from considering the geometry of the ship segments and v_{ai} can be calculated from the conservation of mass flow rate [7]:

$$v_{ei}A_{ei} = v_{ai}A_{ai}. (5)$$

The time taken to fill this segment is finally calculated using:

$$\Delta t_i = \frac{z_{ei} - H_{(i-1)}}{v_{ei}}. (6)$$

The new mass used (representing the mass of the ship plus water inside) once a segment is filled is:

$$m_{Si} = m_{S(i-1)} + \Delta m_s, (7)$$

where Δm_S is given in equation (3). Subsequently from calculating the new volume of water in each segment, the process can be repeated appropriately to find all the values of Δt_i, and finally sum them to find the total time for the ship to fill, and so sink:

$$T = \sum_1^N \Delta t_i, (8)$$

where N segments are used in total.

Example of Calculation using the Algorithm
[8] A ship of mass m_{S0} = 500 imperial tons, r_s = 6.5m (approximately) and water velocity v_{e1} = 5ms^{-1} (an assumed measured value of original water flow into the perforation site) with N = 5, and using the average velocity into and out of a segment in equation (7) gives T = 25 minutes (approximately).

Conclusion
A time to sink of T = 25 minutes is certainly not unreasonable, thus confirming that this could be a potentially useful algorithm, especially when one considers that the times taken for ships to sink are well known to be of the order of tens of minutes. This algorithm would of course be subject to further developments, such as taking account of more complicated ship geometries for more accurate calculations, perhaps done with a computer program for more iterations of N, to make the calculation as accurate as possible. One would also again have to take account of the mass of objects (including people) on board the ships, which should be added to the original mass of the ships before sinking, especially as the ship would likely to be abandoned by people along with lifeboats which are heavy, etc. These would need to be accounted for in more accurate calculations.

References
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