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P2_8 The Tide is High but I am still Short

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Abstract

This work considers the acceleration due to tidal forces on Earth due to the Moon and due to Earth and the amount it would stretch a human's cortical bones. It is found that a human's change in height is negligible and therefore cannot be measured. It is estimated that in order for humans to feel tidal accelerations due to breaking bones, they would have to be at a distance of 151 km from a Neutron star of mass 3.98×10^{30} kg.

Introduction

Tidal forces due to the gravitational pull of the Moon and the Sun can be observed in terms of rising and falling sea levels on Earth day to day. Some people claim that because the human body is mainly made out of water they can feel the tidal pull of the Moon. This paper considers the tidal acceleration gradient exerted by various celestial bodies on the human body and the amount it can stretch the cortical bones.

Tidal Forces Stretching the Limbs

The difference in the gravitational forces exerted by a primary body on one side of a secondary symmetrical body, and its centre of mass, is considered to be what creates the tidal effects and is called the tidal force [1]. If we consider a human at the Earth's surface, the tidal force is the gradient of the gravitational force exerted by the Moon on the human's head compared with the human's centre of mass. The tidal force, ΔF, can be found by differentiating Newton's law,

$$\frac{dF}{dr} = \frac{2GMm}{r^3} \approx \frac{\Delta F}{\Delta r}. \quad (1)$$

Although there is a direction to the force, the vector notation and negative signs have been omitted, as the tidal force is acting towards the primary body (or away from it at the other end), which is exerting the tidal force.

Assuming $r >> \Delta r$, the differentials can be approximated to be differences. Equation (1) applies to a body, symmetrical about the axis along which the force acts (longitudinal), with mass, m (assumed to be 75.0 kg), of length $2\Delta r$ along the direction of the force (taken to be 1.70 m for a human), and with a distance of r between its centre of mass (on Earth's surface) and the centre of the Moon. Our human, for mathematical purposes is approximated to be cylindrical. For this case, r is 3.78×10^8 m [2], M is the mass of the Moon, 7.35×10^{22} kg [2] and G is the gravitational constant. For all cases considered in this article, the human and both the primary and secondary body are in line, such that the gravitational pull on the human is along its length (head to toe). Equation (1) was rearranged, to give the tidal force on the human, exerted by the Moon, F_T,

$$F_T = \frac{2GmM\,\Delta r}{r^3}. \quad (2)$$

In order to work out the amount that this would stretch the body, the Young's modulus [3], Y, has to be considered, mathematically given by

$$Y = \frac{F_T\,L_0}{\Delta L\,A_0}, \quad (3)$$

where Y is the Young's modulus, F_T is the stretching tidal force, L_0 is the initial length, which is $2\Delta r$, ΔL is the length the body is stretched by and A_0 is the original cross-sectional area of the cylindrical body, taken to be approximately 126×10^{-3} m^2 for the cylindrically modelled human with a radius of 20.0 cm. Substituting (2) into (3) and rearranging for ΔL gives the length the body is stretched by due to the tidal forces,

$$\Delta L = \frac{L_0 GmM}{r^3 Y\, A_0}. \qquad (4)$$

If we consider Y of the cortical bone, the stiffest part of the human body, to be 1.90×10^{10} Pa [4], using (4) the amount the bones would be stretched by is approximately, 8.20×10^{-21} m.

The Earth itself will provide an acceleration gradient across the body and therefore a tidal force as well. Using 5.97×10^{24} kg, the mass of the Earth, M, and 6.38×10^{6} m, the radius of the Earth, r, ΔL due to the Earth would be 1.40×10^{-13} m, which is considerably more than the stretching due to the tidal forces of the Moon [2]. Adding these two changes in length together gives the total stretching, which is approximately 1.40×10^{-13} m, as the stretching due to the Moon's tidal forces is minimal.

Tidal Stretching in Different Places

Jupiter's Moon Io is known to experience very large tidal forces due to Jupiter, which pull and squish Io enough to heat its insides to a point where volcanic activity is large [2]. If the model human considered earlier becomes an astronaut and is moved to Io's surface, where he experiences the tidal forces of Jupiter (M is now the mass of Jupiter, 1.90×10^{27} kg, and r is the distance from Jupiter to Io's surface, 4.24×10^{8} m [2]), which would stretch the human by approximately 1.50×10^{-16} m. As for the Earth, Io itself will be exerting a tidal force on the body. Using equation (4) with 8.93×10^{22} kg as the mass, M, and 182×10^{4} m, the radius, r, of Io [2], these would stretch the human cortical bones by approximately 8.90×10^{-14} m. This gives the total stretching, as Io's pull is much more substantial than Jupiter's.

Assuming that the cortical bone is brittle, such that the ultimate strength is the stress required to fracture the bone, the closest the human body could get to an arbitrary neutron star was worked out. For the bones to break, the ultimate strength for our astronaut has to be larger than the tidal force per cross-sectional area perpendicular to the direction of the force, A_0. Combining this with (2), the equation can be rearranged to give the closest distance the astronaut can get to the neutron star without bone fracture occurring. Mathematically this can be expressed as

$$r > \sqrt[3]{\frac{2GMm\,\Delta r}{A_0\, U}}, \qquad (5)$$

where U is the ultimate strength, which we have taken to be a general value of 78.8×10^{6} Pa, according to [5].

Considering a massive Neutron Star of mass, M, 3.98×10^{30} kg [6], this distance, r can be calculated to be approximately 151 km, which would put the astronaut in a very hostile place.

Conclusion

Our estimates have shown that the tidal force on Earth cannot be felt by a human, as the tidal force and the amount it stretches the human bones is negligible. The values calculated for Earth and Io are in addition to the compression by the normal force. As this force is constant, the compressed cylinder is considered the starting point for the calculations.

In order for bones in the human body to break due to the tidal force exerted by a celestial body, one would have to get closer to a Neutron star than humanity has ever achieved.

The gravitational forces exerted by other bodies have been neglected and for a more detailed calculation, the exact force components and the human's relative length on Earth and Io could be taken into account. A more accurate value of the Young's modulus could be obtained for further work as well.

References

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