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P1_5 Relativistic Optics Strikes Back

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Abstract

This paper expands upon previous work on relativistic optics by investigating the radiation pressure exerted on a vessel travelling at relativistic speeds toward a Sun-like star. It is concluded that a power of 3 x 10^{18} W is required to keep the vessel moving at γ=1000 against the radiation pressure, 4 orders of magnitude less than the energy needed to move at this speed initially. Further work should consider all other sources of radiation in the vessel's field of view.

Introduction

Previous work on relativistic optics investigated what an interstellar traveller moving at relativistic velocities would observe [1], with reference to the depiction of this scenario in the 1970s motion picture *Star Wars*. Here we expand upon this work by considering how the impinging radiation from a nearby star in the field of view of an interstellar traveller would affect their motion, due to these relativistic effects. Following the nomenclature of the previous study [1] the traveller moves at velocity v = 0.9999995c toward the impinging radiation source, giving γ = 1000.

A wayward journey to the Sun

For the purposes of this discussion, we assume that the *Millennium Falcon* has accidentally journeyed into our solar system and is on a direct course for the Sun as it passes the Earth, such that d = 1.496x10^{11}m (1AU) (assuming the flux of radiation from all other sources will be outweighed by the flux from a nearby star). Figure 1 shows the angular region into which the solar flux is aberrated, defined by α' as calculated in [1]. Observation of figure 2 of [1] allows us to identify the region of this aberrated radiation which behaves approximately as an isotropic source (since the aberration is angular dependent, the aberrated source is no longer isotropic). This angle is given by α'= 0.95mrads. The ratio of the angle in the Sun's frame, α, to the total initial angle (π) gives the portion of the total luminosity contained in α'; 48% of the total solar luminosity. Now the problem can be treated with the conventional inverse square law in spherical coordinates.

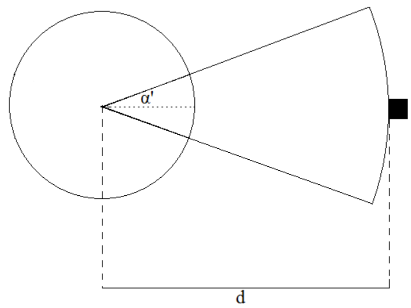

Figure 1: The diagram shows the aberrated solar flux impinging onto the surface of a vessel travelling through the solar system at d=1AU.

The Luminosity of the Sun in the frame of the traveller is given by [2],

$$L_{\odot}' = \sigma A_s {T'}^4 (0.48), \tag{1}$$

where $L_{\odot}'$ is defined by the blackbody temperature shift $T' = \gamma T(1+\beta)$ (calculated using equation 3 in [1], where the Doppler shift actually has a dependence on $\cos\alpha$. This dependence has been neglected since it only causes a factor of 2 change in T'), σ is the Stefan-Boltzmann constant (5.67x10^{-8}Wm^{-2}K^{-4}), and A_s is the surface area of this aberrated region. The reader may notice that the solution of the planck law for the radiation in the observer's frame will not yield the same solid angle relation, given it's the blackbody temperature's angular dependence. It is important to realise that if we consider just the radiation in the total solid angle to be evenly distributed but account for the aberration in the temperature shift, we can just choose a small area for the emitted radiation, which gives an approximate measure of the local luminosity. The factor of 0.48 represents the fraction of the solar luminosity being considered. This transformed blackbody temperature fully characterises the intensity of radiation seen by the vessel (it includes all the effects of transforming frames; time dilation, aberration, Doppler shift, and increased photon arrival rate). The blackbody temperature of the Sun in its rest frame is given by $T = 5780$K [2]. We need the flux of this Doppler shifted/aberrated radiation at a distance d such that the resultant radiation pressure on the vessel can be calculated. This flux is given by

$$S' = \frac{L_\odot{}'}{\int dA}, \qquad (2)$$

where the integral $\int dA$ is performed over the spherical cap of solar flux at distance d. However, since the solid angle into which this radiation is emitted only decreases with the radial distance from the centre of the Sun, when we take the ratio of A_s with $\int dA$ we find it just gives the ratio of $R_\odot{}^2$, the radius of the Sun, with d^2. Equation (2) becomes

$$S' = \sigma \frac{R_\odot{}^2}{d^2} T'^4 (0.48) \qquad (3)$$

The relation for radiation pressure is given by [2]

$$P_{rad} = \frac{S'}{c}(1 + R), \qquad (4)$$

where c is the speed of light and R is the reflectivity of the surface onto which the radiation is impinging. Since the intensity of solar radiation peaks in the X-ray range (λ'=0.20–0.35nm [1]), we assume here that the *Millennium Falcon* is equipped with a photometric filter, such that this radiation is reflected, whilst visible light transmits. Without this assumption, radiation pressures are not trivial to calculate. This means $R \sim 1$. Thus, the power required to counteract this force is given by

$$P_t = \frac{v}{c} S' A_v (1 + R), \qquad (5)$$

where A_v is the surface area of the vessel and is ~ 34.37m x 8.27m, based on the *Millennium Falcon* [3].

Using equation (3), the solar flux S' =6 x 10^{15} Wm^{-2}; this gives a power of P_t =3 x 10^{18} W. The kinetic energy, *KE*, required to reach $\gamma = 1000$ initially is given by

$$KE = mc^2(\gamma - 1). \qquad (6)$$

Assuming a lower limit mass of 1000kg, the kinetic energy to reach $\gamma = 1000$ initially is 9.0×10^{22}J. Therefore travelling for 1s in the field of view of this radiation requires $\sim 10^4$ times less energy than that required to reach γ=1000 initially.

Conclusion

It is concluded that any advanced civilisation capable of interstellar travel at γ = 1000 would need to consider the radiation pressure exerted on the vessel in question and the resulting power required to maintain such a gamma factor, although it would not prevent travel at these speeds given the energy calculation.

Also, in this model only the radiation from the Sun has been considered. Taking into account all other forms of radiation in the field of view adds further dimensions to the problem (the number density of stars in the field of view, and their respective magnitudes and distances, and the Cosmic Microwave Background Radiation (CMBR)). This would result in further power requirements for the vessel, and thus we suggest this as possible further work.

References

[1]Argyle J, Connors R, Dexter K, Scoular C, *P1_3 Relativistic Optics,* PST **11**, (2012 – In progress)

[2]Tipler P, Mosca G, *Physics for Scientists and Engineers,* 6th edition, 2008, W.H. Freeman and Company,

[3]http://starwars.wikia.com/wiki/Millennium_Falcon (accessed on 13/11/2012)