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P4_3 Total Free Fall

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Abstract

This paper analyses the logistics of The Fall, a fictional elevator through the Earth in the 2012 film *Total Recall*. In the film, it is claimed that the elevator can make its journey in 17 minutes by free falling through the centre of the planet. By modelling the actual free fall of an object in a vacuum through the Earth, it is shown that this trip would in fact take over 42 minutes. Drag forces were then considered, showing that without an external force The Fall would enter damped harmonic motion and would never reach its destination.

Introduction

In the 2012 film *Total Recall*, there is an elevator the size of a 30 storey skyscraper running through the centre of the Earth, linking Britain with Australia [1]. As the rest of the world is uninhabitable due to chemical warfare, The Fall transports people through the core of the planet in a claimed 17 minutes via the force of gravity.

Discussion

The effective gravitational acceleration, a_{eff}, can be represented as,

$$a_{eff} = \frac{GM(r)}{r^2}, \tag{1}$$

in a radial system within a massive object, where G is the Gravitational constant and $M(r)$ is the Earth's mass within the radius r [2]. Approximating the Earth as being uniformly dense, this can be simplified to,

$$a_{eff} = g_{surf}\frac{r}{R_{Earth}}, \tag{2}$$

where g_{surf} is the acceleration due to gravity on the Earth's surface, 9.8ms^{-2} and R_{Earth} is the radius of the Earth. If this is the only force acting on the elevator, then it can be thought of as a harmonic oscillator. This would have a period T given by,

$$T = 2\pi\sqrt{\frac{R_{Earth}}{g_{surf}}}. \tag{3}$$

This gives a period of motion of 84 minutes to The Fall. If it was stopped after falling from Britain to Australia, it would take half this time at just over 42 minutes.

Now, if the elevator shaft is not a vacuum, then air resistance will need to be considered. If this is the case, then the drag force,

$$F_D = -\frac{1}{2}\rho v^2 C_D A, \tag{4}$$

will have to be calculated. Here ρ is the density of the medium through which they are travelling, v is the velocity of the elevator, C_D is the drag coefficient and A is the surface area of the elevator.

In an interview with the director, it was stated that The Fall was effectively a 30 storey skyscraper [3]. By looking at the mass of the Empire State Building for comparison, The Fall was estimated to have a mass of around 120,000 tonnes [4] and the roof area was approximated to be equal to roughly 120m by 40m from freeze frames in the film [1]. The Fall also has a roughly flat roof and floor giving it a drag coefficient of 2.1 [5].

For simplicity, this paper models that the initial stages of free fall are unopposed by drag, but the drag force is considered to come into effect after dropping through 1km of the Earth. By analysis of equation 2, the velocity after 1km of unopposed free fall through the Earth was found to be $140ms^{-1}$.

At this point, the gravitational force on the elevator is $1.18x10^9$N. The drag force here is of magnitude $1.21x10^8$N, meaning that the resultant force accelerating the elevator has weakened by approximately 10%. As the elevator falls closer to the centre of the Earth, the gravitational force will weaken and the drag force will increase in accordance with equation 4. This will further slow the motion of The Fall through the planet.

It can therefore be seen that this will lead to damped harmonic motion of those within The Fall; without an external force acting against the drag, its occupants would oscillate with ever decreasing amplitude about the Earth's core.

Conclusion

From the analysis above, it is clear that The Fall is not capable of achieving what is claimed in the film. In order to even reach the other side of the planet, it will need an external power source other than gravity.

Furthermore, if it were to travel the 12,756km journey in 17 minutes, this would require an average speed of over Mach 36. To model this more accurately would require analysis of objects travelling through fluids much faster than the speed of sound if this research were to be continued.

Another element of The Fall that would have to be examined is its heat shielding; by passing through the core of the Earth, it would need to be able to cope with extreme temperature changes in a short space of time. This would likely be a concern for the longevity of the elevator over multiple journeys as it expands and contracts with the heat changes.

References

[1] *Total Recall,* 2012, motion picture, Columbia Films, USA,

[2] Tipler P.A & Mosca, *Physics for Scientists and Engineers, Fourth Edition, Page 343* Freeman & co,

[3] *http://io9.com/5931362/why-an-elevator-through-the-center-of-the-earth-made-total-sense-for-total-recall*, accessed on 27/10/12,

[4]*http://www.newyorktransportation.com/info/empirefact2.html*, accessed on 30/10/12,

[5] *http://physics.info/drag/*, accessed on 30/10/12.