Journal of Physics Special Topics

A3_3 Piercing water

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5th November, 2012

Abstract

This paper investigates under what conditions grains of sand are able to penetrate the surface of a body of water. It is found that there are a range of particle sizes for which grains of sand will not be able to penetrate the surface on impact, even when travelling in the highest speed winds on the globe. The analysis used can also be generalised to any particles of known density and any fluid with known surface tension.

Introduction

Particles carried by atmospheric circulation will often come into close contact with bodies of water, such as the sea or lakes etc. This paper will look at the requirements for such particles to penetrate the water by overcoming its surface tension.

Discussion

Firstly, some assumptions are made about the grain that is impacting on the water's surface. It is assumed to be a spherical body of uniform density with an initial kinetic energy directed towards the surface. The particle impacts on the surface of the water, which deforms around it as shown in figure 1. The surface tension is assumed to be overcome when the particle reaches a depth below the surface, h=2R. At this point it is below the original position of the surface and the water can close around the particle.

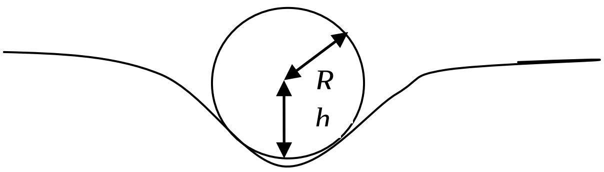

Figure 1

The net force on the particle is then given by:

$$F = F_W - F_S, (1)$$

where F_W=mg is the weight of the particle and F_S is the force due to the surface tension of the water.

$$F_S = \sigma L, (2)$$

where σ=0.0728Nm^{-1}[2] is the surface tension of water at STP and L the length of arc in contact with the water. Geometric analysis gives:

$$L = 2R \times \arccos\left(\frac{R-h}{R}\right). (3)$$

The minimum energy required to stop the particle is then given by:

$$-\int_0^{2R} F.dh = \text{initial kinetic energy}. (4)$$

Evaluating the integral:

$$-\int_0^{2R} F.dh = 2R \times (\sigma\pi R - mg). (5)$$

Using the assumption for the grain to be a spherical body of uniform density, equation (5) becomes:

$$-\int_0^{2R} F.dh = 2R^2 \times \left(\sigma\pi - \frac{4}{3}\pi R^2 \rho g\right). (6)$$

This equation therefore gives the minimum kinetic energy required for a particle to penetrate the surface for a given particle radius, R, and is plotted in figure2, with the density being that of a sand grain, ρ=2648kgm^{-3}[3].

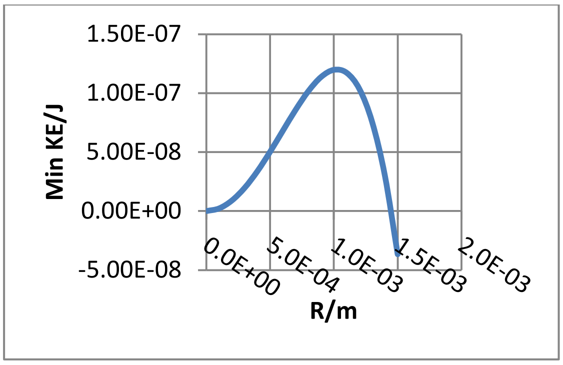

Figure 2: Plot of equation (6), Min KE vs particle radius, R

This means that at small *R* there is a minimum initial kinetic energy that a particle must have to penetrate the surface. This becomes larger for larger particles up to a point, and then the weight of the particle becomes the dominant factor for the largest particles. These have no minimum required kinetic energy as the weight exceeds the surface tension force, even at rest.

The particle radius with the highest minimum kinetic energy is found by differentiating the function of equation (6) with respect to *R* and setting it equal to 0:

$$0 = 4R \times \left(\sigma\pi - \frac{8}{3}\pi R^2 \rho g\right). (7)$$

Solving for $R \neq 0$,

$$R = \sqrt{\frac{3\sigma}{8\rho g}} = 1.02 \times 10^{-3}\text{m}, (8)$$

for sand impacting on water. Inputting this into equation (6) gives a minimum energy of $1.2\text{x}10^{-7}$J.

It can be reasonably assumed that all particles in a fluid flow have the same energy as they will all be colliding with one another and exchanging energy via these collisions. This means that a wind speed can be calculated that may give these larger particles enough energy to penetrate a body of water:

$$\text{wind speed} = \sqrt{\frac{2 \times \text{min. Energy}}{m_{air}}}, (9)$$

where m_{air}=mass of a single air particle in kg.

Taking an air particle to be that of N_2; the main component in Earth's atmosphere, m_{air}=28amu=$4.65\text{x}10^{-26}$kg[4]. This gives a minimum wind speed of $2.27\text{x}10^{9}\text{ms}^{-1}$ for sand grains in the wind to penetrate the surface of the water. This is clearly not feasible as it is greater than the speed of light so it is likely that there exists a cut-off point, beyond which particles could not penetrate the surface.

Taking the highest wind speed recorded: 408km/h=113ms^{-1}[1], an energy of $2.99\text{x}10^{-22}$J can be calculated by rearranging equation (9). By using this energy as the work done in equation (6), it can be seen by solving as a quadratic for R^2 that the corresponding radii will be relatively close to the radii at which the minimum energy is 0; the discriminant will be at:

$$\Delta = 4\sigma^2\pi^2 - \frac{16}{3}\pi\rho g \times \text{min. Energy}. (10)$$

The second argument for the above energy will be much smaller than the first. This means that there will be 2 solutions to the quadratic at points very close to where the minimum energy is zero. A very accurate calculation would be needed to determine these points but from fig. 2 it can be seen that the 2nd point will be approximately *R*=0.0014.

Conclusion

The analysis shows that there will be a range of grain radii, from the micron scale up to approximately 1.4mm, for which grains will not penetrate a body of water upon impact. This includes grains travelling in winds up to the highest wind speeds reached on Earth.

If the particle density and fluid surface tension are known, this method can be used to analyse the impact of any particle upon a body of any fluid. It will be found that for any combination which produces a sufficiently high energy from equation (6) for a range of values of *R*, there will be a range of particle sizes which will be unable to penetrate the fluid, even at very high wind speeds.

References

[1]http://www.wmo.int/pages/mediacentre/infonotes/info_58_en.html (Accessed 04/11/12)
[2]http://www.engineeringtoolbox.com/water-surface-tension-d_597.html (Accessed 04/11/12)
[3]Skinner B.J., Appleman D.E. (1963), Melanophlogite, a cubic polymorph of silica, American Mineralogist 48: 854–867.
[4]http://www.chemicalelements.com/elements/n.html