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P4_15 Home Comforts

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Abstract

This paper considers modifications a conventional terrestrial toilet design may require in order to ensure a successful flush if it was placed on the surface of Mars. It is found that the water tank must be located a factor of 2.6 times higher than it would be on Earth to compensate for reduced gravity on the Martian surface.

Introduction

Since the dawn of human space flight, humans have dreamt of one day colonising Mars. A martian environment may be difficult to become accustomed to, and would be made easier with a few home comforts. Hence, this paper considers modifications a conventional terrestrial toilet design may require in order to ensure functionality on Mars.

Toilet on Earth

Before the Mars design can be scrutinised, a working terrestrial toilet must first be considered. This is best done in the context of the Bernoulli equation, adapted for this scenario:

$$\frac{P_{in}}{\rho}+\frac{v_{in}^2}{2}+gh_{in}=\frac{P_{out}}{\rho}+\frac{v_{out}^2}{2}+gh_{out}\,, \qquad (1)$$

in which P is the pressure on the fluid, ρ is the density of the fluid, v is the velocity of the fluid, h is the height of the fluid and g is the acceleration due to gravity. The left hand side of the equation takes the subscript 'in', denoting water entering the toilet pan, and the right hand side takes the subscript 'out', denoting water leaving the system.

P, and ρ cancel, as they remain unchanged both before and after the toilet has been flushed, reducing the equation to:

$$\frac{v_{in}^2}{2}+gh_{in}=\frac{v_{out}^2}{2}+gh_{out} \qquad (2)$$

h_{in}, the water level in the pan, will be set at zero, and h_{out}, the outflow level in the pipes below the toilet, will be assigned it's height in relation to h_{in}. For a terrestrial system this would give h_{out} a value of approximately -0.5 m.

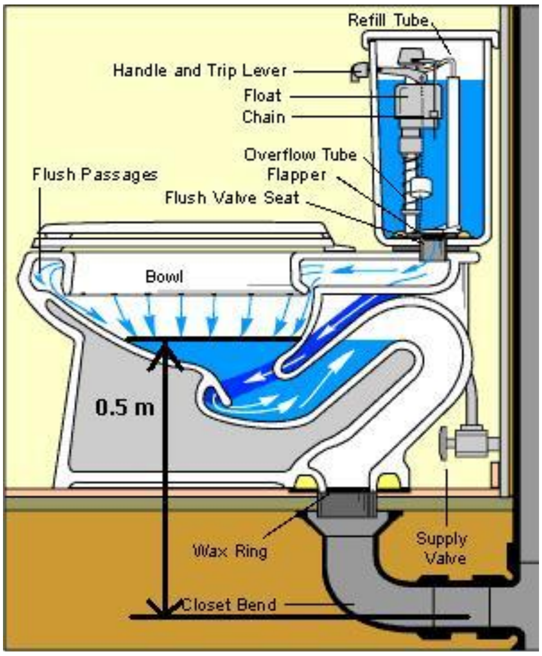

Figure 1: Schematic diagram of a toilet.

Using this equation and an approximation of V_{out} (the velocity of the water leaving the system en route to a sewage management system), an estimate of V_{in} (the velocity of the water entering the toilet pan) can be made, if equation (2) is rearranged for V_{in}:

$$v_{in}=\sqrt{v_{out}^2+2g(h_{out}-h_{in})}. \qquad (3)$$

The approximation of V_{out} can be made using the law of mass conservation:

$$A_1V_1 = A_2V_2, \qquad (4)$$

and the definition of flow rate:

$$Q = AV, \qquad (5)$$

in which A is the cross sectional area of the pipe containing the fluid, V is the flow velocity and Q is the flow rate.

An average flush takes approximately 5s and uses approximately 10 litres of water (equal to 0.01 m^3). Hence the flow rate Q out is approximately $2\text{x}10^{-3}$ m^3s^{-1}.

Solving equation 5 for V, and substituting Q and the area of the outlet pipe A (assigned an estimated radius of 0.04m [2]) gives an outflow velocity V_{out} of 0.4 ms^{-1}.

Substituting this value into equation (3) along with the estimate of h_{out} gives a; V_{in} of 2.9 ms^{-1}.

Toilet on Mars

This value represents an input velocity capable of clearing the pan of any deposits, and will hold on Mars as it does on Earth. Thus, the height of the tank required to give this input velocity on both Earth and Mars can now be analysed.

The law of conservation of energy can be utilized to calculate the required tank height. The water stored in the tank has gravitational potential energy given by:

$$U = mgh \qquad (5)$$

Assuming this is converted wholly into kinetic energy as it falls to the pan the following relationship holds:

$$\frac{1}{2}mv_{in}^2 = mgh \qquad (6)$$

Solving for h:

$$h = \frac{v_{in}^2}{2g} \qquad (7)$$

This equation can be used to establish tank height requirements given that a V_{in} of 2.9 ms^{-1} is considered a successful input velocity for the water: on Earth, g is equal to 9.8 ms^{-2} and hence h is equal to 0.42 m above the pan. On Mars, g is equal to 3.7 ms^{-2} [3] and hence h is equal to 1.1 m above the pan.

Conclusion

It can be seen that there are no major changes that need to be made to the design of the toilet for it to remain functional on Mars. The reduced gravity can be accounted for by elevating the water tank by a factor of 2.6 (h_{Earth}/h_{Mars}), the same as the ratio of the gravitational forces. This increase in height is not significant, and is therefore unlikely to affect martian building design.

In spite of this, the conventional toilet design is unlikely to ever appear on the surface of Mars given the scarcity of water. Water will be a precious resource. Hence, it is far more likely that a design based on pressurised air flow will be employed.

References

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