Journal of Special Topics

A3_2 Auroral Toroid

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October 25, 2011

Abstract
This paper investigates the possibility of modelling the Earth's auroral ring current network as a large toroid surrounding the Earth's own intrinsic magnetic field. Values for the electron density are found to be of the same order of magnitude as the current accepted values using this simple model.

Introduction

The auroral ring current system is created by three sets of motion of charged particles. The first is a bounce motion (between magnetic poles), drift motion (along the circumference of the Earth) and then gyro motion, which is the oscillatory motion of a particle around a magnetic field line. The combination of these motions results in a toroid shaped current region. Estimating a toroid, the electron density is approximated at a radius of 6.6R$_E$ and a non-relativistic energy of 1KeV [1].

Electron Density

The magnetic field within a toroid of radius *r* is given as [2];

$$B_T = \frac{\mu_0 N I}{2\pi r}, \tag{1}$$

where μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ N/A²), *I* is the current through the toroid (A) and *N* is the number of turns in the toroid.

Current is defined as the charge flowing (*q*) per unit time. The expression for time can be replaced by the drift time [3],

$$T_D = \frac{2qr^2 B_E}{W} \quad , \tag{2}$$

which is effectively the period of time for an electron of energy *W* (Kev), to circumnavigate the Earth at a particular radius, giving a new expression for the magnetic field within the toroid;

$$B_T = \frac{\mu_0 N W}{4\pi r^3 B_E}. \tag{3}$$

r is the radius from the centre of the Earth to the midpoint of the toroid and B_E is the magnetic field at the Earth's equator.

Obviously, one cannot possibly count the number of turns as would be possible in a conventional current loop system. However, by combining the frequency of gyrations (the gyrofrequency, $\Omega = qB_E/m_e$) around a magnetic field line (with one complete gyroradius being equal to one turn) with the time taken to orbit the Earth, the number of 'turns' can be expressed as;

$$f \times T_D = \frac{qB_E}{2\pi m_e}\frac{2qr^2}{W} = \frac{q^2 B^2{}_E\, r^2}{\pi W m_e}. \tag{4}$$

Here *f* is the related frequency ($f=\Omega/2\pi$) [4] of a charged particle (s^{-1}) and m_e is the mass (kg) of an electron (9.11×10^{-31} kg).

Substituting equation (4) into equation (3) gives the magnetic field within the toroid (created by one electron),

$$B_T = \frac{\mu_0\, q^2\, B}{r 4\pi^2 m_e}. \tag{5}$$

Equation (5) contains constants and variables that are easily measured. Furthermore, the magnetic field of the Earth at 6.6R$_E$ can be given as

$$B = \frac{B_0}{({}^{r}/_{R_E})^3}, \tag{6}$$

where B_0 is the magnetic field at the Earth's equator (3×10^{-5} T), R_E is the radius of the Earth and r has been previously mentioned (in this case r is equal to 42,000km).
Using accepted values for the charge and mass of an electron, the magnetic field inside a toroid created by one oscillating electron is given to be 2.2×10^{-30} T.

Currently accepted values for the magnetic field induced by a ring current (when very active) is given by the Dst index as approximately 100nT [5]. Using this value for the total ring current magnetic field and the field induced by one charge, the number of electrons within the toroid is estimated to be approximately 4.5×10^{22} particles. To calculate the particle density, the volume of the toroid must be known. The volume of a toroid is given to be [6];

$$V_T = 2\pi^2 L r^2, \tag{7}$$

where L is the length of the toroid around the Earth (given by the simple equation for the circumference of a circle, in this case L is equal to 264×10^6 m). The radius of the toroid (equal to the gyroradius) is calculated as;

$$R_C = \frac{m_e v}{qB}, \tag{8}$$

Where B is the aforementioned magnetic field at a certain radius (as seen in equation (6)), v is the velocity of the electron which is a function of its energy, in this case 1 KeV correlates to 18.7×10^6m/s. This gives a gyroradius of approximately 1.06km. Substituting this value into equation (7) gives the toroid volume to be 5.9×10^{15} m³. This corresponds to a particle density of 7.6×10^6 m^{-3} or 7.6 cm^{-3}.

Conclusion

The method of modelling the ring current system on a simple toroid has both its successes and failures. Using a value for the electron energy as 1KeV gives a realistic volume for the volume of the ring current as generated by electron motion [7]. Furthermore, this method also demonstrates that the contribution to the magnetic flux by the electrons is small and that the density of the electrons at this radius is also much smaller in magnitude than that found in the ionosphere (~10^7 cm^{-3}). However, the calculated density is seven times the measured density for the ring current (~1cm^{-3}) [7]. This is partly due to the fact that a median energy has been used and does not represent higher energies of the order of MeV or colder plasma (eV). Additionally, the model does not take into account the contribution of protons which actually contribute the most towards the ring currents flux (~80%) [8]. Lastly, this model neglects to treat the ring current as a collection of plasma and so ignores magnetohydrodynamic effects and to a lesser effect, kinetic effects. This coupled with the fact that the ring's field is constantly changing via the feed of plasma into and out of the magnetosphere and the magnetotail (thus distorting its shape), indicates that if the ring current is to be modelled as a toroid, more influencing factors must be considered to accommodate for its complex structure.

References

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