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A4_2 Sand Timers: A Hydrodynamical Approach

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Abstract
This paper examines the relationship between the volume of sand and the time measured by a sand timer. The problem was approached by simplifying the shape of a sand timer and assuming the sand can be described by hydrodynamics. The results showed that the total time measured is dependent on the dimensions of the sand timer.

Introduction

Sand timers operate by measuring the time taken for a certain volume of sand to fall from one section through a small hole to a lower section. This paper aims to examine the relationship between the volume of sand contained in a sand timer and the time it measures.

Modelling the flow of sand

In order to model how the sand flows with respect to time it is necessary to make some simplifying assumptions. The primary assumption is to treat the sand as a fluid. Both dry sand and liquids similarly display continuum behaviour, which refers to materials behaving macroscopically, as opposed to the dynamics of individual particles [1]. Sand behaves in this way due to air voids between grains, which keeps them apart and reduces friction [2]. This allows the sand to flow freely in a similar way to fluids, thus justifying the initial assumption. By treating the sand as a fluid, it is possible to use fluid dynamics to find the time taken for a volume of sand to pass through the sand timer.

It will also be necessary to simplify the shape of the sand timer. This paper considers a sand timer consisting of two conical sections, of radius *r* and height *H*, joined by a small cylinder of diameter *w*, where *w* is significantly smaller than both *r* and *H*. This shape, illustrated in figure 1, will allow the sand to fall from the upper cone, through the gap, to the lower cone. This model assumes that the volume of sand completely fills the upper cone, and that the volume cut off at the tip of the cone by the gap is negligible.

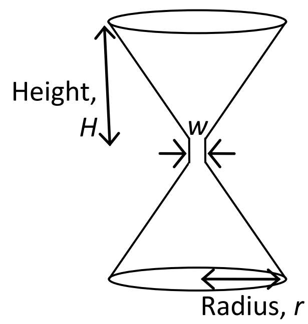

Figure 1. A diagram showing the simplified shape of a sand timer.

In order to find the time taken for the sand to flow from the upper cone, the volume flow rate needs to be determined. The volume flow rate, I_V, is defined below in equation (1) [3], where *A* is the cross-sectional area of the gap, and *v* is the speed of the sand at the gap.

$$I_V = Av. \qquad (1)$$

By substituting for the cross-sectional area, the volume flow rate can be shown to be:

$$I_V = \frac{\pi}{4} w^2 v. \qquad (2)$$

In order to determine I_V, the speed of the sand at the gap is needed. This is found using Bernoulli's equation, shown in equation (3) [4], where *P* is the pressure, *ρ* is the density, *g* is the acceleration due to gravity, and *h* is the height of the sand relative to the gap. The value of *h* will initially be equal to the height of the sand timer, *H*, and will decrease with time until it is zero. Bernoulli's equation applies for any incompressible fluid under steady flow, which must be the assumed conditions for the sand.

$$\frac{1}{2}v^2 + \frac{P}{\rho} + gh = \text{constant.} \qquad (3)$$

This equation is used to find the speed of the sand at the gap by equating the parameters of the sand at the top of the sand timer (height *h*, approximately *v*=0) to the sand falling through the gap (*h*=0). It is assumed that the pressure, density and acceleration due to gravity are constant throughout the system. Therefore, by using equation (3), it can be shown that the speed of sand falling through the gap is:

$$v = \sqrt{2gh}. \qquad (4)$$

Now that the speed of the sand has been determined, equation (4) can be substituted into equation (2), to find the volume flow rate, which represents the rate of volume of sand, *V*, passing through the gap with respect to time, *t*:

$$I_V = \frac{dV}{dt} = \frac{\pi}{4} w^2 \sqrt{2gh}. \qquad (5)$$

The total time taken for the sand to fall through the gap, *Δt*, can be determined from equation (5). This is done by rearranging for the time element, *dt*, and integrating over the total time and total volume. For simplicity, cylindrical coordinates can be used when integrating over the volume. Therefore, the total time measured by a sand timer with these dimensions is found to be:

$$\Delta t = 8\left(\frac{r}{w}\right)^2 \sqrt{\frac{H}{2g}}. \qquad (6)$$

Discussion

The results show that the total time measured increases with the square of the radius and the square root of the height of the cone containing the sand. Equation (6) also shows that the total time is inversely proportional to the width of the gap squared.

This is as expected, as an increase in the total volume would result in more sand having to pass through the gap, thus increasing the total time. It is also intuitive that an increase in the gap width would decrease the time, as more sand will be passing through per second.

The result also shows a dependence of the total time measured on the acceleration due to gravity, *g*. However, changes in *g* are considered negligibly small to be significant.

Conclusion

This paper has found a relationship, between the dimensions of a sand timer, which correspond to the volume of sand contained, and the total time that it measures.

Although this method has considered a simplified shape consisting of two cones, the volume integration could be altered for different shapes. Further investigations could include analysing how the time measured varies with shapes of sand timers.

The method used to analyse the flow of sand has made a major assumption that the sand can be modelled using fluid dynamics. Although the sand particles can be approximated to display fluid-like behaviour, shear forces and velocity gradients present in the sand imply that the flow of sand differs from the flow of liquids [5]. This analysis could be improved by using an alternative technique, for example using a corresponding hydrodynamic description for the flow of granular material [6].

References

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