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A3_11 "Take the Next Left …" at Olympus Mons

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Abstract

This paper analyses the effects due to special and general relativity on a GPS satellite orbiting at an altitude of 3 Mars radii. It was found that in order to keep both the ground station and the satellite in synchronisation the clock on the satellite would have to be 1.56 µs per Martian day faster than the one on the ground.

Introduction

Mankind will eventually colonise other planets the first of which is likely to be Mars. Once colonies grow it may be necessary to navigate the Martian surface. This will probably involve the use of a global positioning system (GPS) similar to the technology that is used to navigate on Earth today. In order to give very accurate determination of a persons' position these satellites need to have extremely accurate clocks. These clocks also need to be in synchronisation with ground based clocks. The theory of special and general relativity suggest that the clocks will be out of sync with each other due to the velocity of the satellite and the weaker gravitational field of the planet respectively. This paper will therefore examine the time difference over the course of one Martian day due to these effects.

Special Relativity

The special theory of relativity states that the time it takes for a clock to tick will increase with the velocity of an object, i.e. time will slow down in the reference frame of the object. The relationship between the proper time, and the observed time is given in Eq. 1:

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}, \qquad (1)$$

where t_0 is the proper time, t is the observed time, v is the velocity of the satellite and c is the speed of light which is equal to $3\text{x}10^8$ ms^{-1} [1]. The length of a Martian day is 24.6597 hrs [2], this is equivalent to $8.88\text{x}10^4$ s. Using this value for t_0, the observed time t can be calculated.

The orbital velocity of the satellite can be found from the force due to gravity and the angular acceleration, Eq. 2, and Eq. 3, [1], respectively.

$$F = \frac{GMm}{r^2} = ma, \qquad (2)$$

$$a = \frac{v^2}{r}, \qquad (3)$$

$$\Rightarrow v = \sqrt{\frac{GM}{r}}, \qquad (4)$$

where G is the gravitational constant ($=6.67\text{x}10^{-11}$ $\text{m}^3\text{kg}^{-1}\text{s}^{-2}$, [1]), M is the mass of Mars ($=6.42\text{x}10^{23}$ kg, [2]) and r is the orbital radius, from the centre of Mars. At Earth, GPS satellites orbit at an altitude of 20200 km [3] (~3 Earth radii). Therefore the altitude used for a Mars GPS satellite will be estimated at 3 Mars radii (R_M), where the radius of mars is $3.396\text{x}10^6$ m [2]. This therefore gives a value of r (in Eq. 4) as $4R_M$ ($= 1.36\text{x}10^7$). The orbital velocity of a Martian GPS satellite is then calculated to be 1774 ms^{-1}, using Eq. 4.

The time dilation due to this velocity can be calculated by substituting the above values into Eq. 1. The difference between two clocks, one on the surface and the other on the GPS satellite, can therefore be calculated by subtracting the proper time from the observed time. This gives a value of 1.55 x 10^{-6} seconds. Thus for every Martian day the difference between the clocks will be increasing by 1.55 µs, due to special relativity.

General Relativity

The general theory of relativity predicts that the time it takes a clock to tick will increase with gravity, i.e. time will be slower in a stronger gravitational field. One solution to the Einstein equations of general relativity is the Schwarzschild metric, shown below [4],

$$c^2 d\tau^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} - r^2 d\psi^2, \quad (5)$$

where all the symbols have the same meaning as defined in the special relativity section above, apart from τ which is now the proper time, t is the observed time and $d\Psi$ is the angular displacement term. In order to evaluate the effect due to general relativity, only the satellite will be assumed to be at rest within the gravitational field of Mars. This means that $d\Psi^2$ and dr^2 are both equal to zero. This therefore leaves,

$$c^2 d\tau^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2. \quad (6)$$

Eq. 6 is rearranged to give Eq. 7 below, from which the time dilation effect can be calculated.

$$\tau = \left(1 - \frac{2GM}{c^2 r}\right)^{\frac{1}{2}} t. \quad (7)$$

The time dilation due to the Martian gravitational field can be calculated by substituting the values for *G, M, c, r,* and τ, from the special relativity section, into Eq. 6. The difference between two clocks, one on the surface and the other on the GPS satellite can therefore be calculated as 3.11x10^{-6} seconds. This therefore means that the difference between the ground station and the GPS satellite will increase 3.11 µs every Martian day.

Discussion

The time dilation effects due to special and general relativity counter each other. For a moving satellite time goes slower, however, for a satellite in a weaker gravitational field time goes faster. Therefore the resultant time difference between the GPS satellite and the ground station is 1.56 µs per Martian day in the favour of general relativity.

Conclusion

If mankind were ever to colonise Mars and wished to use GPS satellites to aid navigation around the planet the clocks on the satellites would have to be set to run 1.56 µs faster than the ground station per Martian day due to the combined effects of Special and General relativity.

References

[1]P. A. Tipler & G. Mosca, *Physics For Scientists and Engineers,* Sixth Edition, (W.H. Freeman and Co., 2008), p378 and pAP-4
[2]http://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html, (15/03/2011)
[3]http://www.kowoma.de/en/gps/orbits.html (15/03/2011)
[4]J. A. Peacock, *Cosmological Physics*, (Cambridge University Press, 1999), p51.