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A3_8 Two mirrors and infinity

D.Karazhov, L.Ryan, T.Booth

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

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Abstract

We work through a scenario of an observation of multiple images when two parallel mirrors are facing each other, with an omnidirectional light source situated between them. We find that a typical conventional light bulb as a source shall initiate approximately 83 images perceivable by human eye.

Introduction

Two mirrors facing each other have a potential to create many images of an object. Some of us, when standing between such mirrors, may have asked ourselves a question: do the mirrors initiate an infinite number of images.

With basic knowledge of physics concepts we can immediately answer no to that question. The radiation power emitted by the object is finite, and so is the number of photons emitted per unit time, thus the intensity will eventually reach exactly zero.

In this article we are interested in how the images decrease in intensity, and also what is the maximum number of images an average human eye would be able to visually perceive in such a scenario.

Model

Figure 1 illustrates how an observer between such two mirrors will detect many images of the object, upon observation of one of the mirrors.

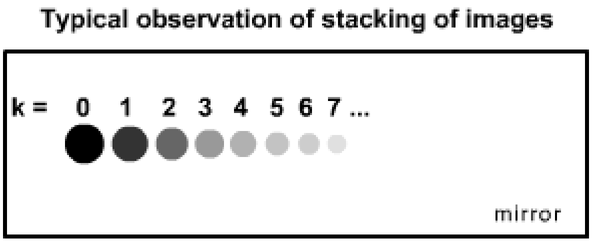

Figure 1: Images seen by an observer when looking at one of the mirrors as described in the scenario. *k* identifies the rank of each image.

Note how both, the intensity and the angular size of each consecutive image decrease.

Figure 2 shows schematic arrangement of the light rays responsible for the images.

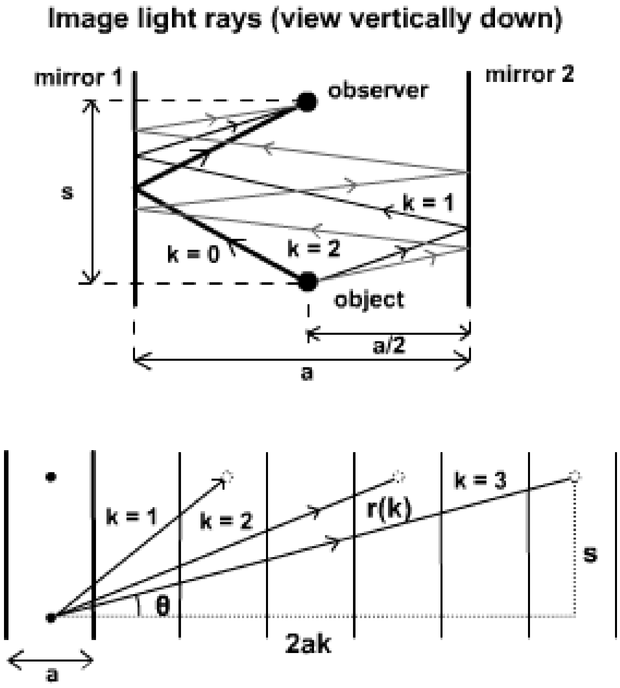

Figure 2: a)(top) Paths of the light rays which create images up to rank 2; b)(bottom) Geometrical representation, which is physically equivalent to representation in (a).

Figure 2(a) is a convenient representation of the optical paths in the scenario. From 2(b) we deduce that r is the distance the light ray travels to the observer, which is a function of image rank k. If we assume that the object is an omnidirectional source, then such a beam obeys the inverse square law [8]:

$$I = \frac{P_0}{4\pi r^2} \, , \tag{1}$$

where I is intensity of the image observed, and P_0 is visible radiation power emitted by the object.

Using Pythagoras' Law we may find the distance travelled as a function of rank:

$$r^2(k) = 4a^2k^2 + s^2 \quad . \tag{2}$$

There are two major mechanisms of intensity loss: inverse square law of $r(k)$ and the loss due to mirror inefficiency. The latter is called reflectivity R, which is defined as the ratio of the first order reflected beam intensity to incident beam intensity. Each time a reflection occurs, the loss may be accounted for by reducing the source power from P_0 to RP_0. We may easily deduce that for rank k there are $2k$ reflections. Consequently, the initial power should be reduced to:

$$P_0 \to R^{2k} P_0 \quad . \tag{3}$$

Thus, substituting (2) and (3) into (1) yields:

$$I(k) = \frac{R^{2k} P_0}{4\pi(4a^2k^2 + s^2)} \quad . \tag{4}$$

In order to estimate the maximum rank of an image observable by a human eye, we shall assume that the object is a typical domestic light bulb and is the only source of visible radiation. Retrieving values for typical mirror reflectivity (94%) from [1] (averaged over optical wavelengths) and efficiency if a tungsten light bulb (2.5%) from [2] defined as the ratio of emitted power of visible radiation to electrical power supplied, we may find $P_0 = 1.5$ W for a 60 W light bulb. We assume realistic values for a = 3 m and s = 2 m, the yielding intensity $I(k)$ as a function of rank is plotted in figure 3.

To estimate lowest visually perceived by human eye image intensity we shall use the widely known fact that the smallest observed apparent magnitude of a star is 6 [7]. The intensity/magnitude relationship can be obtained from many astronomy books, e.g. [3]:

$$I_2 = I_1 10^{0.4(m_1 - m_2)} \quad . \tag{4}$$

Taking m_2 = 6, and using Vega as a reference star with m_1= 0.03 [4], $I_1 = 10 \times 10^{-9}$ W/m^2 (using inverse square law with luminosity [5] and distance [6]), we find $I_{min} = I_2 = 4 \times 10^{-11}$ W/m^2.
Substituting the estimated values into equation (4) and solving it numerically, we find that the rank ≈ 83.

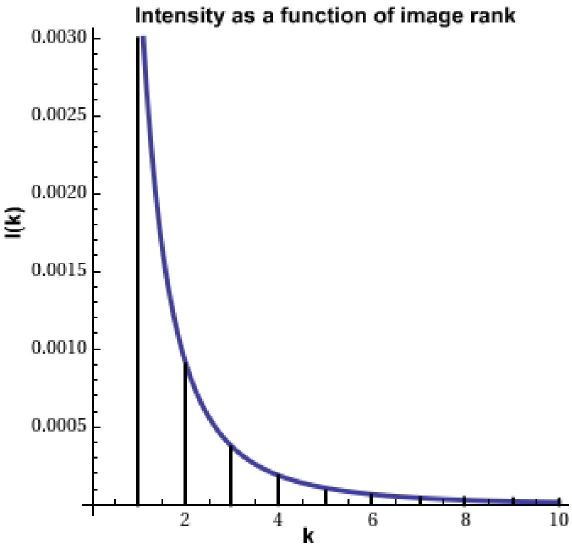

Figure 3: Intensity plot as a function of image rank for an omnidirectional light source of initial power 60 W. Vertical lines indicate allowed values of k.

Conclusion

It was estimated that for a typical arrangement of the two mirror problem, a human eye can detect up to 83 images, provided that it is dark in the room and the source is an omnidirectional light bulb. The 80th image is expected to appear as a dim star.

However, many factors, which have a potential altering our conclusion, have been omitted. For example, a tungsten light bulb is not a spherical source, but rather has a shape of the tungsten wire. Also, the reflectivity of a mirror depends on the angle of incidence, which changes with rank k.

We can nevertheless with certainly conclude that the number of images shall never be infinite in such an arrangement, and that only few dozens of images can be detected by a human eye.

References

[1] http://www.frontsurfacemirror.org/specs.htm (1/3/11)
[2] http://www.energy4you.net/aboutcfl.htm (1/3/11)
[3] Binney and Merrifield, Galactic Dynamics, Princeton University Press, 1998, p. 48
[4] http://simbad.u-strasbg.fr/simbad/sim-id?Ident=Vega (1/3/11)
[5] http://lanl.arxiv.org/abs/astro-ph/0603327 (1/3/11)
[6]http://www.astronomyfactbook.com/constellations/lyra.htm (1/3/11)
[7]http://en.wikipedia.org/wiki/Apparent_magnitude (11/3/11)
[8] P. Tipler, Physics for Scientists and Engineers, Fourth Edition, p. 455.