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A4_5 As the Crow Flies or as the Mole Digs?

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Abstract

It is a well-known fact the shortest path to follow is that of a straight line. This paper takes that concept to the extreme and considers the possibility that we could construct a straight tunnel from one place on the Earth to another and looks at the reduction in transport time we could expect from such a transportation system. It is calculated that the travel time to connect any two points on the Earth via a straight tunnel through the Earth is only dependant on the density of the Earth and this travel time is calculated to be 42.2minutes. Using the deepest hole that has been dug as a limit, it is found that the equivalent of 792km across the surface of the planet can be travelled via the tunnel. Because it only takes 42 minutes to travel this distance it is found to be quicker than taking an aircraft.

Introduction

As civilisation has advanced we have looked for quicker and quicker modes of transport, this paper looks at the concept of shortest path length. A common phrase when regards to travel and distance is "as the crow flies", this statement essentially encompasses the fact that the shortest possible route between two locations is a straight line. This paper looks at the concept of a "gravity train" this idea was originally conceived in a letter from Robert Hooke to Isaac Newton in the 17th Century. The gravity train connects any two points on the Earth's surface by a straight tunnel and the traveller can then travel through this tunnel to his destination. This paper uses the mathematical solution of this problem and looks at whether a tunnel built using this concept would be a quicker method of travel then conventional means. The maximum depth that mankind has drilled to will be used as a constraint on the system as it can be considered that materials within out possession would be capable of withstanding the conditions they were subjected to at that depth.

Theory

If we assume that it would be possible to build a tunnel that could connect two points on the Earth's surface we could use this tunnel as a method of transportation. Not only would this direct line provide the shortest route but it could be powered by its gravitational attraction to the Earth. If we assume that the tunnel could be made frictionless, in a vacuum for example, we can calculate the force using Newton's equation of Gravitational attraction,

$$F = \frac{GmM(r)}{r^2}, \qquad (1)$$

where *m* is the mass of the transport module, *M(r)* is the mass of the Earth that is closer to the centre of the Earth than the transport module, *G* is the gravitational constant and *r* is the distance to the centre of the Earth. This equation forms the basis of the theory behind the gravity train idea as it is the force that powers the transport. A full mathematical solution can be found on Purdue University's website [1]. This mathematical solution shows the time taken for such a transport to complete its journey to be

$$T = \frac{\pi}{\sqrt{(3/4)\pi\rho G}}. \qquad (2)$$

Where *T* Is the time taken, *ρ* is the density of the medium you're travelling through and *G* is the gravitational constant. We can see that the only variable is the density of the Earth which is ~5500kgm^{-3}[2]. If we put the numbers into Eq.2 we arrive at a travel time for our transport, *T* = 2534s or 42.2 minutes. We can see that any two points connected by a gravity train would take 42.2 minutes to travel between assuming that no energy was lost to friction. In order to see whether this would be a viable replacement for travel we will look at the maximum depth that has been drilled to. Using that as a baseline for the maximum depth of our tunnel we can calculate the maximum distance we could connect using a gravity train and compare that to existing transport method. We can see the geometry of the proposed situation in figure 1.

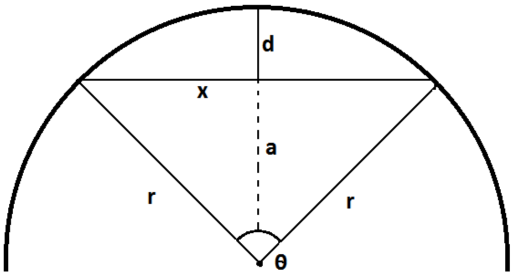

Figure 1 - Sketch showing the geometry of the proposed gravity train. Where *x* is the tunnel the train follows, *r* is the radius of the Earth, *d* is the maximum depth drilled to and *ϑ* is the angle of separation.

In order to make the comparisons, we shall calculate the maximum angular separation that can be achieved by the tunnel at our value for the deepest we have dug. Using trigonometry we can deduce that $a = r\cos\left(\frac{\theta}{2}\right)$ where $a = r - d$. We can rearrange this to give us a value for θ which can be seen in eq.3

$$\theta = 2\cos^{-1}\left(\frac{r-d}{r}\right). \qquad (3)$$

The deepest hole ever drilled is reported to be the Kola Superdeep Borehole which reached a depth, *d*, of 12,261m [3]. Using this number we can calculate an angular separation, $\theta = 7.11^\circ$. In order to convert this angle into the equivalent distance across the surface of the Earth, we divide it by 360^0 and multiply it by the Earth's circumference. This gives a distance of 792 km as the equivalent distance across the surface of the Earth, the arc length, that is covered by our gravity train tunnel.

Conclusion

It is found that transport that operated as the described gravity train would take 42.2 minutes to connect any two points. Using the deepest tunnel mankind has dug we put a limit on the depth and thus the distance covered by our tunnel, under this limit it was found that the tunnel could connect two points that were separated by 792km on the surface of the Earth. This distance is approximately the distance between London and Hamburg. The proposed gravity train can make the journey in 42.2 minutes whereas conventional aircraft take around 90 minutes [4]. This shows that travel by gravity train would be quicker than current travel options.

References

[1] http://www.math.purdue.edu/~eremenko/dvi/gravsol.pdf

[2] http://hypertextbook.com/facts/2000/KatherineMalfucci.shtml

[3] http://www.icdp-online.org/front_content.php?idcat=695

[4]http://www.flightstats.com/go/FlightStatus/flightStatusByFlight.do;jsessionid=359990DCDC970C2F451EA18EFD221142.web3:8009?airlineCode=BD&flightNumber=3299&departureDate=null