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A4_6 The Penny Drops

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Abstract

A British 1p coin is dropped from a great height. Its terminal velocity is calculated assuming that it falls face-down, alongside the force (and subsequently pressure) it would exert upon a surface on landing. The value is compared to the tensile strength of normal human skin in order to examine the effects it might have if it landed on a person, and is found not to be high enough to break the skin.

Introduction

A common piece of folklore imagines someone standing atop a tall skyscraper (commonly, the Empire State Building) and dropping a penny. This penny then collides with a pedestrian below, killing them instantly. This paper aims to consider whether this really is the outcome of such a scenario by considering the properties of a penny and the physics of its fall to calculate the force (and the pressure) inflicted on human skin when the penny lands on its victim.

Terminal Velocity

When an object falls through the air, it has two forces acting on it. The first is the force due to gravity,

$$F_{\mathrm{g}} = mg, \tag{1}$$

where m is the mass and g is the acceleration due to gravity (on Earth, $g = 9.81$ ms^{-2}).

The second force that has an effect on the penny is the drag due to the fluid

$$F_{\mathrm{d}} = \frac{C_{\mathrm{d}}\rho_a A v^2}{2}, \tag{2}$$

where C_{d} is the coefficient of drag; ρ_a is the density of air; A is the area of a penny and v is the velocity of the object moving. In this paper, it will be assumed that the penny will fall face-down and as such the area of the penny $A = \pi r^2$ (the case in which the penny falls with its edge facing the ground is left for further investigation).

Eq. (2) shows that the force due to drag on the penny will increase as it accelerates until the gravity and drag forces are equal, at which point, the terminal velocity v_{t} has been reached. An equation for this velocity is derived by equating Eq. (1) and Eq. (2):

$$mg = \frac{C_{\mathrm{d}}\rho_a \pi r^2 v_{\mathrm{t}}^2}{2} \Rightarrow v_{\mathrm{t}} = \sqrt{\frac{2mg}{C_{\mathrm{d}}\rho_a \pi r^2}}. \tag{3}$$

Landing

An expression for the deceleration a of the penny on landing can be derived using the terminal velocity of the penny v_{t} from Eq. (3). Starting with the equations of motion, bearing in mind that the initial velocity $u = v_{\mathrm{t}}$ and the final velocity $v = 0$ ms^{-1}, and also assuming a constant deceleration on impact,

$$v = u + at \tag{4}$$

$$v = 0 \Rightarrow u = v_{\mathrm{t}} = -at \tag{5}$$

$$\Rightarrow -a = \frac{v_{\mathrm{t}}}{t} \tag{6}$$

where t is the time it would take the penny to decelerate upon hitting the skin. Using the relationship between deceleration a and impact force F_{i}, we can derive an equation to find this force and an equation to find the pressure exerted on impact P (where A_{i} is the area of the penny that impacts upon the skin):

$$F_{\mathrm{i}} = ma \Rightarrow P_{\mathrm{i}} = \frac{ma}{A_{\mathrm{i}}} = \frac{mv_{\mathrm{t}}}{tA_{\mathrm{i}}} \tag{7}$$

Discussion

The above equations assume knowledge of several constants, which are as follows. The mass m, diameter $2r$ and thickness s of a British 1p coin are given by [1] as $m = 3.56 \times 10^{-3}$ kg, $2r = 2.03 \times 10^{-2}$ m and $s = 1.52 \times 10^{-3}$ m respectively. The drag coefficient of a thin disc $C_{\mathrm{d}} = 1.1$ [2] and the density of air $\rho_a = 1.29$ kg m^{-3} [3].

Using these values for the constants, a terminal velocity is derived from Eq. (3) as $v_{\mathrm{t}} = 12.3$ ms^{-1}. This value can be put into Eq. (6) and then Eq. (7) to calculate values for the impact force F_{i} and the pressure exerted P_{i}.

However, the time taken for the penny to decelerate t must be assumed. Since it is assumed that the impact would be visible on a high-speed camera (which typically record at 1000 fps [4]) and since it is desirable to minimise the time taken to decelerate to provide the highest possible force and pressure exerted on the skin, a value of $t = 10$ ms is assumed. The impact area of the penny with the skin A_i is also assumed (in this paper, $A_\mathrm{i} = s^2 = 2.31 \times 10^{-6}$ m^2 is used).

In this scenario, it is found that $F_\mathrm{i} = 4.37$ N and $P_\mathrm{i} = 1.89$ MPa.

Conclusion

It is known that the average tensile strength of human skin $P = 20.89 \pm 4.11$ MPa [5]. The calculated value for the pressure exerted on the person upon landing, $P_\mathrm{i} = 1.89$ MPa, is much lower than this value. The penny's impact with its victim would not even break the skin.

REFERENCES

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