What's in a grade? The real meaning of mathematics grades at GCSE and A-Level

Gerry Walton
Rednock Comprehensive School
Dursley
Gloucestershire
GL11 4BY

Dudley E Shallcross*
Bristol ChemLabS
School of Chemistry
Bristol University
BS8 1TS

***d.e.shallcross@bris.ac.uk**

Abstract
The scheme of work in mathematics and science subjects at GCSE and A-Level has been constantly changing over the last fifteen years. Under the auspices of a pilot scheme funded by Chemistry for our Future (CFOF) we review the current scheme of work in mathematics at both GCSE and A-Level from the three main examining boards and provide insight into what mathematical skills one might expect from a student entering a Physical Science degree programme, in particular in Chemistry.

A-Level Mathematics (Background information)
Prior to 2000 students traditionally followed A-Level courses which were:

either	Pure Maths 50%	+	Applied (Mechanics)	50%
or	Pure Maths 50%	+	Statistics	50%

The depth of study was relatively intense with some fairly rigorous calculus and formal proofs. For example in Statistics students were expected to be able to derive the Normal Distribution mean and standard deviation using calculus learnt within the Pure section of the course. Whilst all students received a solid grounding in Pure Mathematics they usually studied only either Mechanics or Statistics (albeit to some depth).

Post 2000 the new A-Level system for all subjects came into force based upon 6 modules for the full A-Level. Structurally it was as follows:

Pure Maths 50%	Applied Modules 50%
3 modules P1, P2, P3	3 from Stats, Mechanics or Discrete

The advantage of the new system was that whilst retaining most of the traditional Pure content it enabled students to broaden their Applied content (albeit at a cost of depth). However, it quickly became apparent that students were finding the new structure and content difficult. Results dipped dramatically and exam boards responded by reviewing the whole structure again and temporarily offering an extra examination session in November for students wishing to retake modules. Mathematics was the only subject forced to do this. The current structure is now as follows: (see appendix for detailed content)

Core Pure Maths	66.6%	Applied units	33.3%
4 modules C1, C2, C3, C4		Combination of any two S1, S2, M1, M2, D1, D2	

Students studying Further Mathematics have considerable freedom in their choice of Applied units but it is compulsory to take the Further Pure 1 module. The current A-Level system is again under review with a move to a four module system. However, Mathematics and the Science subjects are not included in this restructuring. We expect the six modules to be with us for some time yet.

Background information: GCSE Mathematics
Whilst changes to the A-Level system have been fundamental the subject content and level of achievement for GCSE Mathematics (at least at the Higher Level) has remained more or less static. However, structural changes have been made.

2008	First examination of the new two tier GCSE (Higher and Foundation). Currently Maths is the only subject operating at three tiers.
2008	Last examination involving coursework. Currently coursework comprises 20% of the overall total.
2010	A 'Functional Maths' element is proposed to be incorporated into the GCSE. Pilots are on-going at the present time.

Whilst changes to the A-Level system have been fundamental the subject content and level of achievement for GCSE Mathematics ... has remained more or less static.

Gerry Walton is Head of Mathematics at Rednock comprehensive school and has over 25 years experience in teaching secondary school level mathematics.

Dudley Shallcross is Professor of Atmospheric Chemistry and has taught supplementary mathematics courses in the School of Chemistry at Bristol University.

Note: Mathematics is the only subject where students have to work at a level above GCSE in order to gain an A* grade for their coursework.

GCSE Subject Content
Content for GCSE Higher Level courses is virtually identical across the various examination boards. The main differences occur in the delivery of the course which may involve one examination period, or the modular approach with students taking individual modules at set periods during the year.

It is also common to see differences in approach towards coursework with some boards offering set problems which can be assessed either by teachers or by the board. Some options allow schools to set their own coursework tasks which are then marked against set criteria. The new non-coursework GCSE which comes into operation in 2008 retains the content of the previous GCSE, but is to incorporate questions within the examination which directly assess the old coursework elements such as data handling and investigations.

Mathematics is the only subject where students have to work at a level above GCSE in order to gain an A grade for their coursework.*

Students applying for University degrees with an element of mathematics involved would have needed to have followed the Higher Level course at GCSE.
Students following an Intermediate level course (2007 is the last year anyhow!) will not have the depth of algebraic knowledge required. Very occasionally students have successfully taken an A-Level in mathematics having followed the Intermediate GCSE course, but it is a rarity and they have had to work hard at upgrading their skills.

Summary of Higher Level content **relevant** to a Chemistry Course

Solving numerical Problems
Percentages
Surds
Standard Form
Exponential growth

Ratio and proportion
Powers, fractional, inverse

Equations, formulas and identities
Transposition of formula
Factors, linear and quadratic
Index notation
Solving equations
approx solutions
Formula substitutions
Simultaneous equations
Quadratics, use of formula + completing the square

Sequences, Functions, Graphs
Coordinate geometry: use of Y=mX + c, parallel and perpendicular lines
Interpreting graphical information
Graphical solution to equations
Graphical transformations of functions
Vectors: basic definitions + vector geometry
Trigonometrical graphs + basic trig functions including use of Sine, Cosine rules
Bearings, Simple Loci problems.

In addition, the data handling section of the course containing elements of basic statistics, probability and simple data analysis will be included.

Assessment of student capabilities at A-Level

AS-Level

Students entering university with AS-Level mathematics often polarise into two categories:
a) A strong student who started 4 or more A-Level courses but dropped mathematics at the end of year 12 to concentrate on the other subjects. Likely to have obtained a strong grade.
b) A (mathematically) weaker student who managed to take their maths knowledge a little further than GCSE but have now reached their limits.

Grade A/B candidate
Likely to fall in the category (a). A candidate with good all round algebraic skills and able to take on new concepts quickly. However, there will be a lack of depth in areas such as calculus and trigonometry. They will be familiar with the concepts of differentiation and integration but only at a basic level.

Grade C candidate
Algebraic skills much less well developed and often these students have obtained the majority of their marks from the Applied section of the course. No great depth to their Calculus knowledge and students engaging upon a university course containing a substantial mathematical element will initially struggle.

Grade D/E candidate
As above but to a greater degree. The algebraic skills are likely to be fairly basic and this student would have to work very hard to keep pace with the demands of any mathematical content.

A-Level

Grade A candidate
Obviously strong mathematically with good overall skills. Their algebra and calculus in particular will be a strength and there will be a depth of understanding. Students will have a clear knowledge of Algebra courses at university, except for the sections on matrices, complex numbers and some elements of expansions.

Grades B & C
Mathematically quite good students but likely to have lower capabilities at the top end of the A-Level Core content. For example some of the more difficult work on trigonometry and integrations requiring a substitution will be at the upper end of their abilities. Students obtaining these grades at A-Level often have very good marks for their Applied modules e.g. Statistics or Discrete maths and high marks for the first two Core modules. The more in depth Core 3 and Core 4 modules are less well attempted.

Grades D & E
Generally students obtaining these grades will have found much of the Pure content of the A-Level quite demanding. Quite often their marks will have been boosted by solid performances within the Applied modules and a relatively easy Core 1 module. They will have struggled with the later Core 2, 3, 4 modules. Students obtaining a low grade A at GCSE or grades B and C will often fall into this category.

Further Maths
Any student successfully completing the Further Maths Course is by definition a very capable mathematician and well able to cope with the mathematical demands of a university chemistry course.

Scottish entrants
Students following the Scottish Boards syllabus content will have covered similar topics to their English, Welsh, and Northern Ireland counterparts. However, the structure and standard of question papers do differ considerably from the standard A-Level. A rough comparison between syllabus content and standard (at least for mathematics) would be the Scottish Higher qualification to be slightly above the AS-Level. The Scottish Advanced Mathematics course has content which features most aspects of the standard A-Level but also incorporates elements of the Further Maths A-Level.

Any student successfully completing the Further Maths Course is by definition a very capable mathematician and well able to cope with the mathematical demands of a university chemistry course

Assessment of student capabilities at GCSE level
Students applying for university courses where a GCSE is their highest qualification in mathematics will, in the main, not have studied the subject to any great extent for two years. Their mathematical skills will probably have not increased but will have diminished over this period. **It is a strong recommendation that only students taking Higher Level GCSE courses be considered.** Basic calculus and the more advanced algebra is the minimum required for Physical Science degrees at University. For students without higher level GCSE the step up to improve their mathematical knowledge upon entry to University is considerable.

Grade A* Students
These students will have good algebraic skills and be capable of manipulating formulae quite easily. They will not have seen the use of complex numbers or logarithms. Calculus will be almost entirely foreign to them. However, an 'A*' student is mathematically capable and provided sufficient support is given they should be able to upgrade their skills to a sufficient level.

Grade A Students
These students will be competent mathematicians but not intuitive. The range of abilities from just missing out on an 'A*' grade to just scraping an 'A' grade is considerable. It is often the case that students are able to gain an 'A' grade through sheer hard work but they are approaching the limits of their mathematical abilities. The role of coursework is often critical with hard working students able to move into the grade 'A' on the basis of good coursework. An 'A' grade student will have reasonable algebraic skills but may not necessarily be comfortable with the detail required at later stages of the course.

Grade B Students
These students (**which could include Intermediate candidates**) will have a general level of ability in mathematics. For example they will be able to evaluate problems involving basic percentages comfortably, work with standard forms, or solve simple equations. They will not be strong algebraically and they would find topics such as basic calculus very hard. A student, at this level, applying for any University course involving an aspect of mathematics would find the prospect very daunting.

Grade C Students
As for a grade B but to a greater extent. Algebraic skills are likely to be poorly developed. A student with a grade C, will struggle with basic calculus and advanced algebra.

On-going work
As part of a pilot scheme for *Chemistry for our Future* we have reviewed the content of the current schemes of work for GCSE and A-Level mathematics and given indications of the mathematical ability of candidates at each grade. We would be delighted to review the mathematical content of any courses in the Physical Sciences and map grades in mathematics at GCSE and A-Level to them, so that course conveners can determine where the cut offs might be for their supplementary courses and the topics that should be included in them: send a list of the mathematical content to the following e-mail address Karen.Shallcross@bris.ac.uk and entitle the e-mail **ND-CFOF maths**. We are also in the process of surveying mathematics provision and content in Chemistry at tertiary level and will be producing a report on our findings in September 2008.

We would be delighted to review the mathematical content of any courses in the Physical Sciences and map grades in mathematics at GCSE and A-Level to them

Acknowledgments
We thank CFOF under whose auspices this work was carried out. We thank Gareth Price and Simon Bedford at Bath University for their support and useful discussions and Tim Harrison and Karen Shallcross at Bristol University for their contributions to this work.