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Determining Distance or Object Size from a Photograph

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Abstract
The purpose of this paper is to create an equation relating the distance and width of an object in a photograph to a constant when the conditions under which the photograph was taken are known. These conditions are the sensor size of the camera and the resolution and focal length of the picture. The paper will highlight when such an equation would be an accurate prediction of reality, and will conclude with an example calculation.

Introduction

In addition to displaying visual information such as colour, texture, and shape, a photograph may provide size and distance information on subjects. The purpose of this paper is to develop such an equation from basic trigonometric principles when given certain parameters about the photo.

Picture Field of View

First, we will determine an equation for the horizontal field of view of a photograph when given the focal length and width of the sensor.

Consider the top down view of the camera system below (figure 1), consisting of an object (left), lens (centre), and sensor (right). Let d_1 be the distance of object to the lens, d_2 be the focal length, l_1 be the width of the object, l_2 be the width of the sensor, and α be the horizontal field of view of the photograph. Note that the object is wide and close enough such that its image, when in focus, takes up the entire width of the sensor.

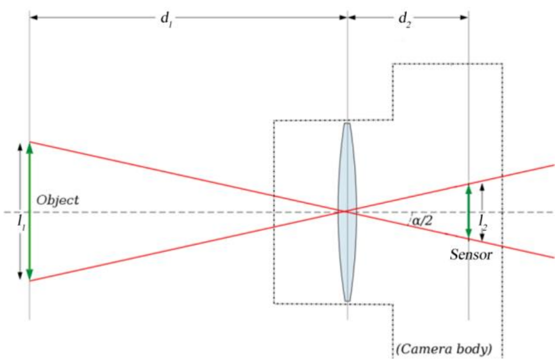

Figure 1 – A simple camera model [1].

On first glance, the relationship between the focal length (d_2) and the horizontal field of view (α) may be expressed as a tan (opposite/adjacent) relationship:

$$\tan 0.5\alpha = \frac{0.5l_2}{d_2} \quad (1.0)$$

$$\alpha = 2\arctan\frac{0.5l_2}{d_2} \quad (1.1)$$

Picture Angular Resolution

With the image dimensions, we can make an equation for one-dimensional angular resolution. Since l_2 describes sensor width and not height, we will calculate horizontal instead of vertical angular resolution:

$$\alpha_{hPixel} = (2\arctan\frac{0.5l_2}{d_2}) \div n_{hPixel} \quad (2.0)$$

$$\alpha_{hPixel} = \frac{2}{n_{hPixel}}\arctan\frac{0.5l_2}{d_2} \quad (2.1)$$

Here, α_{hPixel} is the average horizontal field of view for a single pixel, and n_{hPixel} is the number of pixels that make up a row of the picture.

Object Angular Diameter

If the width of the object on the image is measured, we can develop an equation for the horizontal angular diameter of the object:

$$\alpha_{hObject} = (\frac{2}{n_{hPixel}}\arctan\frac{0.5l_2}{d_2}) \times n_{hpObject} \quad (3.0)$$

$$\alpha_{hObject} = \frac{2n_{hpObject}}{n_{hPixel}}\arctan\frac{0.5l_2}{d_2} \quad (3.1)$$

Here, $\alpha_{hObject}$ is the horizontal angular diameter of the object and $n_{hpObject}$ is the number of pixels in a row that make up the object.

For a sensor constructed such that the vertical and horizontal spacing between pixels are equal, the following formula may be used to measure the angular diameter in any direction:

$$\alpha_{Object} = \frac{2n_{pObject}}{n_{hPixel}} \arctan\frac{0.5l_2}{d_2} \quad (3.2)$$

Here, α_{Object} is the angular diameter, and $n_{pObject}$ is the number of pixels that make up the object in any direction.

Equation 3.0 and 3.2 assume that the angular resolution of a pixel is the same regardless of its position in the picture. In reality, this is impossible due to the planar nature of the camera sensor and the spherical nature of the field of view (figure 2). As camera lenses generally output images with rectilinear projections [2], equations 3.1 and 3.2 are most accurate when the object of interest is offset from the centre such that its constituent pixels have field of views close to the average (equation 2.1 output). The offset is dependent on the focal length.

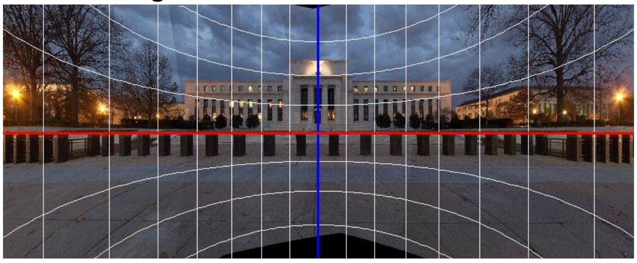

Figure 2 – A rectilinear projection; each square represents the same field of view. Notice how the field of view of an individual pixel decreases towards the edges [3].

Object Distance / Length

An object with a set angular diameter may be an infinite combination of distances and sizes. For example, a further away large object will occupy the same angular diameter as a smaller object at closer distance. Glancing back to figure 1, this relationship may be approximated as:

$$\tan 0.5\alpha_{hObject} = \frac{0.5l_1}{d_1} \quad (4.0)$$

$$\alpha_{hObject} = 2\arctan\frac{0.5l_1}{d_1} \quad (4.1)$$

Combining equations 3.2 and 4.1, we form an equation relating object distance/diameter to the conditions under which a photograph was taken.

$$\arctan\frac{0.5l_1}{d_1} = \frac{n_{pObject}}{n_{hPixel}} \arctan\frac{0.5l_2}{d_2} \quad (4.2)$$

It is important to note that l_1 is in a single plane parallel to the plane of the sensor. Therefore, the equation will be most accurate when applied to an object such that, at all points it occupies in the picture, is at constant distance from the camera.

Example Calculation

For an example calculation, we will be calculating the distance of a Taiga Bean goose from the photographer. The photograph (figure 3) was taken with a Canon T1i, which has a sensor size of 22.3 x 14.9mm [4]. The picture itself has dimensions of 4752 x 3168px and was taken with a focal length of 85.0mm. The height of the goose in the picture is 1302px, and the average Taiga Bean goose has a height of 30.3cm [5]. We assume no distortion, and equal sensor horizontal and vertical pixel spacing.

Figure 3 – A pair of Taiga Bean geese crossing a road. The subject of our calculations is in front [6].

By adding the values into equation 4.2, we arrive at:

$$\arctan\frac{0.5 \times 303mm}{d_1} = \frac{1302px}{3168px} \arctan\frac{0.5 \times 14.9mm}{85.0mm}$$

$$d_1 = 4214.74mm$$

Thus, the goose was 4.21m from the photographer when the picture was taken.

Conclusion

Currently, how well the equation predicts object distance or size is unknown. A method of testing would involve taking a picture of an object where the distance and object size are both known, calculating both values using the equation, and comparing real values with calculated values.

Furthermore, the equation is limited to working with objects that remain a consistent distance from the camera. An improvement would be to account for this, so that the lengths of objects such as roads or very tall buildings from photographs at ground level may be measured.

Another point of improvement is accounting for rectilinear distortion, which currently affects equation accuracy at the centre and edges of a photograph.

Finally, with some further work, an equation may be derived to determine the velocities of blurry objects in a single photograph, or of the same object between two different photographs with the same field of view.

References

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