Journal of Interdisciplinary Science Topics

How Rising Sea Levels May Affect The Length Of Your Day

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12/04/2017

Abstract
The Earth has a near-constant rotational period of approximately 23.934 hours. Rising global temperatures put this period in danger of changing by redistributing polar ice cap mass towards the equator through conservation of angular momentum. The Earth was modeled as a solid sphere with the assumption that melted ice would form a spherical shell over the Earth's surface. We explicitly show that if all ice were to melt, Earth's rotational period would be increased by approximately 0.285 seconds, where the discrepancy with our clocks accumulates to 17.35 minutes after only 10 years.

Introduction

Time-keeping has one of the most complicated and convoluted histories. Because of the lack of relationship between the rotation of Earth and its period of orbit around the Sun, keeping a consistent day length and calendar has proven to be a very difficult task. These difficulties led to the installment of leap years, and the exceptions to these rules, such as the century leap years [1]. While days on Earth are counted by 24-hour clocks, a day only lasts an average of 23.934 hours (86162.4 seconds) [2]. Time's already complex history now faces climate change as an issue, which has been further exacerbated by anthropogenic factors. With global temperatures rising, the planet's ice reserves are constantly melting. Assuming the melted ice is distributed evenly across the globe as a rise in sea level, the Earth's rotational period risks significant change through simple momentum conservation laws.

Theory

It can be assumed that there is no external torque acting on Earth. Therefore, angular momentum is conserved.

$$L_{initial} = L_{final} \quad (1)$$

Using the definition of angular momentum, quation 1 can be rewritten as equation 2 where I is the moment of inertia and ω is the angular velocity [3]. Subscripts i and f indicate 'initial' and 'final' respectively.

$$I_i \omega_i = I_f \omega_f \quad (2)$$

Since we are investigating the period of Earth's rotation, not the angular velocity, this can be rewritten using relation 3.

$$\omega = \frac{2\pi}{T} \quad (3)$$

And subsequently,

$$I_i \frac{2\pi}{T_i} = I_f \frac{2\pi}{T_f} \quad (4)$$

Simplifying and rearranging equation 4 yields equation 5; the period of the Earth after all of the ice on Earth melts.

$$T_f = T_i \frac{I_f}{I_i} \quad (5)$$

In order to determine the moment of inertia, the Earth is modeled as a solid sphere. After the ice has melted, the moment of inertia of the water formed must be considered. It is assumed that the meltwater spreads uniformly over the Earth, essentially forming a spherical shell. Equations 6 and 7 show the moments of inertia of a solid sphere and a spherical shell, where M and R are the mass and radius of the objects respectively [3].

$$I_{sphere} = \frac{2}{5} M R^2 \quad (6)$$

$$I_{shell} = \frac{2}{3} M R^2 \tag{7}$$

Substituting these equations into equation 5 yields equation 8.

$$T_f = T_i \frac{\frac{2}{5} M_{E-ice} R^2 + \frac{2}{3} M_{ice} R^2}{\frac{2}{5} M_E R^2} \tag{8}$$

Simplifying equation 8 yields equation 9, which calculates the period of the Earth after melting.

$$T_f = T_i \left(1 + \frac{2M_{ice}}{3M_E}\right) \tag{9}$$

Earth Mass (M_E)	$5.972 \times 10^{24}\ kg$ [2]
Total Ice Volume on Earth	$3.234 \times 10^{16}\ m^3$[4]
Ice Density	$916.7\ kg/m^3$ [5]
Total Ice Mass on Earth (M_{ice})	$mass = density \times volume$ $m = 2.964 \times 10^{19}\ kg$
Initial Period (T_i)	$86162.4\ s$ [2]

Table 1 – Summary of constants used.

Results

Substituting the values for the mass of the Earth and the mass of the ice (from Table 1) yields the ratio between final and initial period of the Earth.

$$T_f = T_i \left(1 + \frac{2(2.964 \times 10^{19}\ kg)}{3(5.972 \times 10^{24}\ kg)}\right)$$

$$T_f = 1.000003309\ T_i$$

$$T_f = 1.000003309 \times 86162.4\ s$$

$$T_f = 86162.6851\ s$$

$$\therefore T_f - T_i = \Delta T \approx 0.285\ s$$

ΔT is the change in rotational period throughout this shift; approximately 0.285 seconds. Over time, the discrepancy between our 24-hour clocks and the new rotational period accumulates. Using equation 10, this effect is calculated for a period of 10 years (3652.5 days).

$$\frac{0.285\ s}{day} \times 3652.5\ day \times \frac{min}{60\ s} = 17.35\ min \tag{10}$$

Discussion

As shown in the previous section, the change in rotational period can be as significant as 0.285 seconds. The discrepancy between 24-hour clocks and the actual length of day then amounts to 17.35 minutes after 10 years.

Of course, there are ways that civilization has learned to correct time so that it matches the rotation of the Earth. For example, DUT1 is the correction used so that atomic clocks (used for precise time keeping) match the time based on Earth's rotations [6]. The difference in time based on the atomic clocks and Earth's rotation is corrected by introducing a leap second when necessary. By definition, DUT1 must be kept as less than 0.9 seconds. Therefore, the change in period of 0.285 seconds is significant enough that over a few years more leap seconds will be needed to maintain DUT1 as less than 0.9 seconds. This means that using leap seconds will keep the time from becoming as significant as 17.35 minutes [6].

Conclusion

The effects of climate change on sea level rise were analysed to calculate the change in Earth's rotational period due to the change in mass distribution. The change was found to be 0.285 seconds. The effect of this change to the rotation and how it would be dealt with was discussed. These results are further evidence that climate change has significant effects, and that action against this should quickly be taken.

References

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