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How Fast Can a Titan Run?

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Abstract
This paper aims to discuss the limitations on the speed at which an Abnormal Titan from the popular anime *Attack on Titan* can run. Considering their erratic and inefficient running style, and the sheer size of the creatures – which causes a high air resistance – the velocity of the Titan was found to be $100.39\ ms^{-1}$. This is realistic in comparison to how they appear in the anime, and reflects that Titans are exceptionally fast, and thus very dangerous enemies.

Introduction
The Titans (*Kyojin*) are the supposed main antagonists in the manga and anime *Attack on Titan* (*Shingeki no Kyojin*). They are divided into three distinct categories based on appearance and behaviour.

Figure 1 – The Normal Titans from the manga [1] (left) and an Abnormal Titan's first appearance in the anime [2] (right)

The Normal Titans ('Normals', or merely *Kyojin*) are giant humanoid creatures, varying between $3m$ – $15m$ in height, which resemble male humans without visible genetalia as shown in Figure 1. They will chase and devour any humans that cross their path without hesitation – but never move faster than a walking pace [1]. Abnormal Titans ('Abnormals' or *Kikō-shu*) appear the same as Normals – as can also be seen in Figure 1 – but are characterised by their unpredictable actions. These Titans will decide on specific targets to attack, and can perform movements such as running, jumping, and on one occasion even speech [2]. Finally, Titan Shifters ('Shifters') are humans with the ability to transform into Titans. These Titans retain their intelligence and some physical attributes with their human selves, upon transformation [3].

Whilst the Normals and Shifters can be modelled using standard human gait calculations, the erratic nature of the Abnormals' movements mean that they must be considered differently to the other types of Titan. In this paper, a possible model for the gait of an Abnormal will be presented, and thereby a maximum speed for their movement will be calculated.

Basic Gait Model

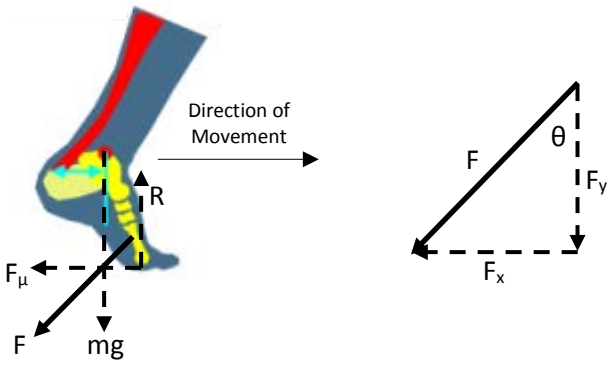

Figure 2 – A simplified model of a foot whilst running. Adapted from the following image: [4] (left) and the components of the downward force exerted by the foot (right)

A gait model is a mathematical description of the particular way that someone walks or runs. This paper will utilise a gait model to calculate the upper limit on the running speed of a fifteen-metre Abnormal.

The left of Figure 2 shows the forces being considered whilst the foot is in contact with the ground mid-run. The forces defined are as follows: the weight of the Titan mg, the reaction force of the ground R, the frictional force F_μ, and the full force exerted as the Titan pushes off from the ground F. On the right, the force exerted in order to push the Titan upwards and forwards is split into its components; the x component F_x, which acts backwards, and the y component F_y, which acts downwards. The angle between the force and its y component is θ, it is assumed to be $60°$ for this model. The larger the angle the larger the x component of the force, and the faster the velocity of the Titan. Thus, the assumumption of $60°$ is used as it is a feasible angle that gives a high velocity.

As can be noticed from the scene in which an Abnormal first appears [5], the gait is both erratic and vastly inefficient. This inefficiency is due to its tendancy to run mostly on the balls of its feet and barely utilising its thigh muscles. Since the pushing action of lifting the heel and the swinging of the leg from the thigh are both significant contributors to fast and efficient movement, this Abnormal's speed is greatly limited. However, since these factors can be ignored, our model is much simpler than the usual model of human gait. Firstly, to find the mass of the Titan the equation below is utilised:

$$m = BMI \times h^2 \quad (1)$$

This equation assumes that body mass index (BMI) remains applicable for humanoids up to fifteen metres tall. Using the commonly-accepted average BMI of $22kg.m^{-2}$ and a height, h, in metres ($15\ m$), it can be calculated that the mass of a fifteen-metre Titan is $4950\ kg$. Following this, using the equation for gravitational force with g as $9.81\ ms^{-2}$ it can be seen that the weight of this Titan is $48.6\ kN$.

In order to determine F, a trigonometric identity is used to relate the vertical component to the total force:

$$F = \frac{F_y}{\cos(\theta)} = \frac{mg}{\cos(60^\circ)} \quad (2)$$

Using the stated values, the total force exerted by the Titan on the ground is $97.1\ kN$. Finally, F_x is the component of F which propels the Titan forward. The x component can be found using another trigonometric identity:

$$F_x = F \times \sin(60^\circ) \quad (3)$$

Therefore this gives a value for F_x of 84.1kN. Assuming that the Titan is running at a constant velocity – i.e. acceleration is zero – then all forces acting upon the Titan must cancel each other out. In this case, F_x (acting forwards) and air resistance will be equal. This allows a velocity to be calculated:

$$F_x = \frac{1}{2} C_D \rho A v^2 \quad (4a)$$

$$v = \sqrt{\frac{2F_x}{C_D \rho A}} \quad (4b)$$

Where C_D is the drag coefficient, ρ is the density of air, and A is the cross sectional area of the titan. The C_D for a humanoid is approximately 0.6, which is assumed to be consistent for the Titans [6]. ρ for air is known to be $1.225\ kg\ m^{-3}$ [7]. Lastly, it was assumed that A for a Titan would be half the body surface area [8, 9]:

$$A = \frac{1}{2} \times \sqrt{\frac{m \times h}{3600}} \quad (5)$$

Using the previously calculated value for m and h as the height in centimetres ($1500\ cm$), the cross sectional area becomes $22.7\ m^2$. Substituting all these values into Equation 4, the velocity of the Titan is found to be $100.39\ ms^{-1}$.

Conclusion

It was found that the fastest speed an Abnormal Titan can run at is $100.39\ ms^{-1}$. This seems to tally with how they are portrayed; in the anime, humans have to ride horses to have a chance of keeping up with the Titans. This is a realistic model; although the air resistance upon a creature this size would be enormous, the amount of force it generates due to its mass in spite of its inefficient running style makes it extremely fast. If they ran like humans or Shifters, these Titans would most likely reach even faster speeds.

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