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Powering Disney's *Frozen* with a Carnot refrigerator

Aaron Goldberg
Honours Integrated Science Program, McMaster University
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Abstract

Frozen is Disney's latest film, in which the character Elsa unleashes winter on her entire kingdom. This paper quantifies the amount of water frozen and the amount of work required by a Carnot refrigerator to do so, arriving at values of $5.49772788 \times 10^{12}$ moles and 5.8×10^{15} Joules, respectively.

Introduction

Frozen is a 2013 award-winning film, featuring the Snow Queen Elsa, who has the ability to create snow and ice from thin air. In the film, she inadvertently freezes the fjord around the capital city Arendelle, plunging the entire kingdom into winter [1]. The present paper discusses the amount of energy a Carnot refrigerator would require to cause this freeze.

Total Amount of Water

The city of Arendelle was inspired by the Norwegian fjord Nærøyfjord (Figure 1) [2].

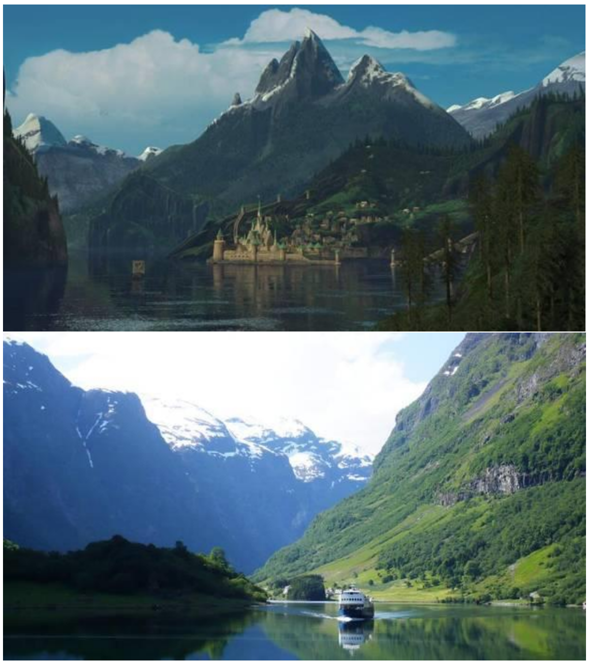

Figure 1: A comparison between the animated city of Arendelle, above, and Nærøyfjord, below [2].

Nærøyfjord is at least 18 km long and 500 m wide [3], and so its surface area is about 9×10^6 m^2. When, in the film, the ice begins to crack, it is apparent that about 1 m of ice is floating on top of the water. Using the density of ice as 0.9167 g mL^{-1} [4] and of freshwater as 1.000 g mL^{-1} [5], the total volume of ice is

$$Surface\ area \times \frac{1\ \mathrm{m}}{1 - \frac{Ice\ density}{Freshwater\ density}} \quad (1)$$
$$= 108043217\ \mathrm{m}^3.$$

Assuming this density of ice, the total mass of ice is 9.9043217×10^{13} g. With the molar mass of water being 18.0153 g mol^{-1} [6], this is equivalent to $5.49772788 \times 10^{12}$ moles of ice.

Isobaric Molar Heat Capacity of Water

To calculate the molar enthalpy change of freezing this amount of water, one must first know how the molar heat capacity of water behaves through the associated temperature range. Heat capacity data for water [7] and ice [8] have been tabulated in Table 1.

Using Microsoft Excel to fit these data to a curve, the equations for isobaric heat capacities of water close to its melting point are

$$\bar{C}_{P,water}(T) = (0.00105345\,T^2 - 0.628043\,T + 168.952)\ \mathrm{J\ mol^{-1}K^{-1}} \quad (2)$$

and

$$\bar{C}_{P,ice}(T) = (-0.000833751\,T^3 + 0.603612\,T^2 - 145.388\,T + 11677.7)\ \mathrm{J\ mol^{-1}K^{-1}} \quad (3).$$

Table 1: Isobaric heat capacity versus temperature for water and ice at 1 atm pressure:

Water Temperature (K)	C_p (J mol^{-1} K^{-1})	Ice Temperature (K)	C_p (J mol^{-1} K^{-1})
273.16	76.014	230.08	25.389
275.16	75.898	236.19	25.527
277.16	75.799	242.40	28.430
279.16	75.713	249.31	28.882
281.16	75.640	256.17	27.577
283.16	75.577	262.81	26.372
285.16	75.523	267.77	18.682
287.16	75.476		
289.16	75.437		
291.16	75.404		
293.16	75.377		
295.16	75.354		
297.16	75.335		

Total Enthalpy Change

Assuming the water transitioned from an ambient 20°C to ice at −15°C, the enthalpy of reaction follows the formula

$$\Delta_{reaction}\bar{H} = \int_{293.15\,K}^{273.15\,K} \bar{C}_{P,water}(T)\,dT + \Delta_{fusion}\bar{H}^{\circ} + \int_{273.15\,K}^{258.15\,K} \bar{C}_{P,ice}(T)\,dT. \quad (4)$$

This enthalpy is the sum of the enthalpy change in liquid water going from 20°C to 0°C, the enthalpy change of water at 0°C freezing into ice, and the enthalpy change of ice going from 0°C to −15°C. Evaluating (4) using (2), (3), and $\Delta_{fusion}\bar{H}^{\circ} = -6008.224\ \mathrm{J\ mol^{-1}}$ [8] leads to $\Delta_{reaction}\bar{H} = -7833.180\ \mathrm{J\ mol^{-1}}$. From the above result for the total amount of water, it follows that $\Delta_{reaction}H = -4.30646921 \times 10^{16}$ J.

Using a Carnot Refrigerator

It is well known that the most efficient heat engine is a Carnot engine, which harnesses the temperature difference between two reservoirs to do work. This can be done in reverse, harnessing work to drive a temperature difference between two reservoirs, and is known as a Carnot refrigerator. In this case, the latter two reservoirs are the ice at −15°C, and the air at 20°C. The coefficient of performance of the refrigerator is the ratio of the heat flow between the two reservoirs and the work input required for such, and follows

$$\frac{T_{cold}}{T_{hot} - T_{cold}} = 7.38. \quad (5)$$

By isolating for work, it can be found that the work required is equal to the total enthalpy change divided by the refrigerator's coefficient of performance. As such, the total mechanical work required by a Carnot refrigerator to power the transformation of the entire fjord from water to ice is 5.8×10^{15} Joules.

Conclusion

It has been shown that in *Frozen*, Elsa froze approximately 5.5×10^{12} moles of water. To accomplish Elsa's feat, a Carnot refrigerator would require 5.8×10^{15} Joules of energy. This amount is equivalent to the energy released by the Hiroshima nuclear bomb 115 times over, or that released by 63 Nagasaki nuclear bombs [9]. This immense number puts Elsa's power into perspective, implying either that the Snow Queen has enormous strength, or that Disney underestimated the ramifications of their animated fantasy.

References

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