Journal of Interdisciplinary Science Topics

Chargrilled Icarus' Wings

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18/03/2013

Abstract
This paper looks at the myth of Icarus and his father Daedalus, specifically the likelihood that the wax in Icarus' wings would have melted during his flight. It has been found that it would have taken between about 42 and 67 minutes for the wax to melt, and the distance from the sun would have played no part; In essence, his father warnings were meaningless and he would have also fallen to his death.

Introduction
In Greek mythology Daedalus built two pairs of wings designed to allow a human to fly so that he and his son could escape from captivity. In the story his son, Icarus, doesn't listen to his warning and flies 'too close to the sun' melting the wax holding his wings together and causing him to plummet into the ocean[1]. In this article it is calculated how long it would have taken the wax in Icarus' wings to have melted, and therefore if they would have done so.

Size of Wings
As the Greek god Zeus had a golden eagle as a personal messenger and as it was considered to be a mighty beast it follows that someone of the age might base a design for a set of wings on such an animal. The rough calculation of the size of wings will therefore come from scaling up those of a golden eagle. In addition to this, eagles are fairly tall upright birds, which is closer to the human body shape than most other birds.

A mid-range height of a golden eagle is about 0.9m[2]. As history tells us, people in the time of Daedalus were shorter than today and, in addition, Icarus was only a young adult. It will therefore be assumed that he would have been about 1.4m tall. The ratio of heights is therefore:

$$\frac{1.4}{0.9} = 1.\dot{5}$$

The median wingspan of a golden eagle is about 2m[2] therefore using this ratio this would mean that a human's wingspan would have to be:

$$2 \cdot 1.\dot{5} = 3.11\ m$$

If we assume that the width of Icarus' shoulders was 0.3m this means that the total width of the wings alone would be 2.81m. As a golden eagle's wings are roughly rectangular and the height of them is about half their total height, the surface area of the wings on a single side would therefore be:

$$2.81 \cdot \left(\frac{0.9}{2}\right) = 1.26\ m^2$$

Incident Energy on the Wings
The radiant power from the Sun can be calculated using the Stefan-Boltzmann Law[3]:

$$P = e\sigma AT^4$$

Equation 1

The sun has a radius of 6.955×10^8m which, using the equation for surface area of a sphere, gives that its total surface area, *A*, is:

$$A = 4 \cdot \pi \cdot (6.955 \times 10^8)^2 = 6.08 \times 10^{18}\ m^2$$

The surface temperature, *T*, of the sun is about 5800K[4] and, assuming that it is a blackbody radiator, it has an emissivity, *e*, of 1. Using the value Stefan-Boltzmann constant, σ:

$$5.6703 \times 10^{-8}\ W/m^2K^4$$

This give the radiant power from the Sun:

$$P = 1 \cdot 5.6703 \times 10^{-8} \cdot 6.08 \times 10^{18} \cdot (5800)^4 = 3.90 \times 10^{26}\ W$$

However, this is the power on the surface of the sun. It is therefore necessary to calculate the intensity of incident energy on the Earth, which can be done using the inverse-square law[5]:

$$I = \frac{P}{4\pi r^2}$$

Equation 2

As the distance of the Earth from the sun is 1.496x10^{11}m the intensity per unit area is therefore:

$$I = \frac{3.9 \times 10^{26}}{4 \cdot \pi \cdot (1.496 \times 10^{11})^2} = 1386.73\ W/m^2$$

As it shall be assumed that Icarus did not leave the atmosphere and it was a clear day with no cloud the amount of this energy absorbed by the atmosphere would be about 16% and the amount reflected would be about 26%[6]. This means that a total of 58% of the incident solar energy would transmit through the atmosphere, equal to:

$$1386.73 \cdot 0.58 = 804.30\ W/m^2$$

This however would be at the equator. Due to the curve of the Earth incident rays would be more spread out moving away from the equator. As Daedalus and Icarus were attempting to escape from Crete, which has latitude of 35.21°, this means that the total incident energy would be:

$$804.30 \cdot \cos(35.21) = 657.15\ W/m^2$$

The radiant energy on the wings would therefore be:

$$657.15 \cdot 1.26 = 828.01\ W$$

Energy Transfer through Feathers

If we model the wings as a white smooth surface then the amount of radiation absorbed by it would be between 25 and 40%[7]. The energy absorbed by the feathers would therefore be between

$$828.01 \cdot 0.25 = 207.00\ W$$

And

$$828.01 \cdot 0.4 = 331.20\ W$$

As the thickness of a feather is so small, it will be assumed that all of this energy is transmitted through the feather and absorbed by the wax.

Energy Needed to Melt Wax

To calculate how much energy would have been needed to melt the wax the following equation can be used[8]:

$$Q = mL_f$$

Equation 3

Where m is the mass and L_f is a constant known as the latent heat of fusion. Assuming that the wax holding the wings together is evenly spread around in a depth of 5mm, in order to hold the wings together and give them the greatest chance of being able to fly, without being too heavy. This then gives a total volume of wax of:

$$0.005 \cdot 1.26 = 6.3 \times 10^{-3}\ m^3$$

The density of wax is about 900kg/m^3[9] meaning that the total mass of the wax would have been:

$$\rho = mV = 6.3 \times 10^{-3} \cdot 900 = 5.67kg$$

The value of L_f of wax is 147kJ/kg[10] so the total energy needed to melt this wax would be:

$$Q = 5.67 \cdot 147 = 833.49\ kJ$$

The energy absorbed by the wax is measured in Watts which is equal to Joules per second. By dividing the energy needed to melt the wax by the energy absorbed this allows us to find the time it would take to melt the wax.

$$\frac{833.49 \times 10^3}{331.20} = 2516.6\ s \cong 41.9\ mins$$

$$\frac{833.49 \times 10^3}{207.00} = 4026.5\ s \cong 67.1\ mins$$

The time taken to melt Icarus' wings would therefore be between 42 and 67 minutes, if it was a clear day.

Conclusion

As the distance from Crete to Athens is about 300km Icarus would have to have been moving at a speed of 335.5km/h, at the very least, to avoid his wings melting on the journey. In addition, it is likely his father's wings would also have melted on the journey and, in fact, the distance from the Sun would not be an important factor.

References

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