

# Journal of Interdisciplinary Science Topics

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## How Balanced is a Liquid Core Dice?

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12/04/2024

### Abstract

Liquid core dice sets have become increasingly popular due to their eye-catching designs adding extra style and physical personalisation to a player's tabletop games. Though all dice makers and retailers state that their liquid core dice are just as balanced as all other dice, many tabletop players would still question this, so this paper aims to investigate the balance of liquid core dice compared to standard resin dice.

**Keywords:** *Board Games; Probability; Dice; Dungeons and Dragons; D20*

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### Introduction

Tabletop Role-playing Games (TTRPGs) use a range of dice to make rolls that aid the storytelling and impact the character's success at various tasks. The results of these rolls can determine whether the character succeeds on an attack, deceives a guard, or spots a trap. In *Dungeons and Dragons (DND)*, the most popular TTRPG, there are 7 different dice used, most frequently the D20, which will be the focus of this paper, where we will be investigating whether a liquid core D20 is balanced and comparing it to a standard resin dice from the same seller.

In gambling, it is understandable why the player would benefit from unbalanced dice, as it would increase their chances of being able to predict the results and helping them win more money. But in TTRPGs, where the roll determines the success of an attack or an interaction, the game will actually be more enjoyable if there is a range of outcomes without any bias. Yes, rolling a 20 in DND will allow you to get the best possible outcome of what you are trying to achieve, but the uncertainty of not always succeeding is what makes it a game rather than just writing a story.

This uncertainty, however, is why players will usually want their dice to be balanced, or they will at least want to make sure their dice aren't biased towards rolling a bad result! Due to the movability and complex structure of liquid core dice, many players have concerns that this would make them less likely to roll evenly.

Most dice sellers will state that they are actually more likely to be balanced than standard mass-produced dice, as they are hand crafted and can be carefully made to ensure there are no air bubbles [1, 2]. With how the liquid spins and shifts, it is still designed to spin evenly, however these reassurances are usually not enough for TTRPG players, who are going to want more certainty before they spend their money on expensive sets.

Submerging a dice in salt water repeatedly and seeing if the same number always floats to the top is a known and tested way of checking the balance of dice [3], but that might not be something you'd want to do with your very expensive dice! This would also only work with dice that are less dense than salt water, as well as having concerns of causing damage, so this is not a fool-proof method.

With such dice, the only way to know for sure if they were even would be to roll the dice repeatedly, recording the results and reducing human error as much as possible, which was the method I used in this experiment.

### Method

The dice tested were bought from the Etsy seller Danieluky [4, 5]; having purchased a liquid core dice and a standard resin dice, so that a non-liquid core dice could also be tested to be used as a control for whether any bias in the dice could instead be a flaw in the sellers crafting process.

I also chose to use a dice tower in this experiment, to avoid any potential bias in uneven rolling by hand. Dice towers are designed to make the dice spin more, and themselves cannot have an effect on the outcome of the roll. A basic 3D printed design from Etsy seller PrintfinityBeyond [6] was used, their Black (Large) option was selected to ensure that the tower would comfortably fit the dice.

The method itself was simple, to roll each dice 4000 times and record the results for analysis. The number was chosen to ensure the sample of results was large enough to get a statistically significant result, while also considering the amount of time it was going to take. The number of times each result was rolled was recorded in groups of 20, to minimise the risk of typing errors in the spreadsheet erasing previously recorded data. There were also a number of things I made sure to do throughout the process to ensure the reliability of the results. Primarily, I was careful with the dice to make sure there was no chance of damaging them or introducing any bias into the experiment: the full details of the steps taken can be found in *Appendix 1* at the end of the paper.

### Results

The full data set of results can be found in Appendix 2 (Liquid Core Dice) and Appendix 3 (Resin Dice), but the trends of the totals of each number rolled can be seen in Figures 1 and 2. As you can see, both dice showed to give unbalanced results, with both rolling a large number more 20s and fewer 16s than the expected mean of 200, among other similar trends of notable values. For a conclusion to be drawn, however, it is important to calculate whether they are just slightly extreme results within expected ranges, or are statistically significant outliers.

A statistically significant outlier is one that is more than 1.5 times the interquartile range (IQR) outside of the first and third quartiles (Q1 and Q3) [7]. Using a calculator, the quartiles can be found such that, for the liquid core dice, Q1 = 184 and Q3 = 214, and for the resin dice, Q1 = 163 and Q3 = 226.5. Therefore, the boundaries that would show a significant outlier can be found to be 139 and 259 for the liquid core dice results and 67.75 and 321.75 for the resin dice set. For the liquid core dice, this would mean that the results of the 16 (126) and 20 (262) would both be considered significant outliers. For the resin dice, this would mean that only the result of the 20 (380) would be considered a significant outlier. This proves that

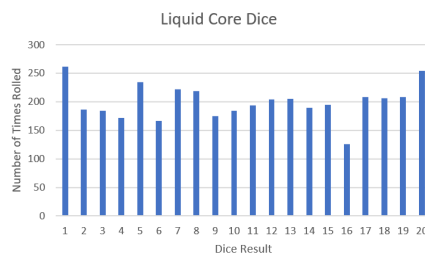


Figure 1 – A graph showing the results from the rolling of the Liquid Core D20.

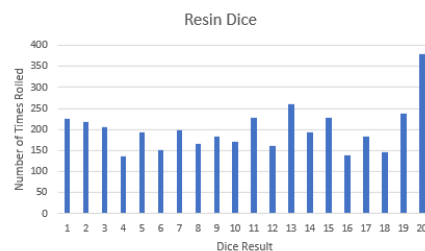


Figure 2 – A graph showing the results from the rolling of the Resin D20.

the unbalance in both dice is statistically significant enough to not be just due to chance; the dice do not roll evenly.

Interestingly, the liquid core dice rolled large numbers of both 20's and 1's, and though only one of the results is a statistical outlier, they are still both noticeable. The two numbers are on opposite sides of the dice (as is the conventional design of all size dice), so the dice clearly is not weighted to one side in a particular way. However, those are the two critical numbers in DND, so it has to be questioned whether there could be some intentional weighting in the dice to make for a more interesting game.

### Conclusion

The liquid core dice tested showed to be statistically unbalanced, however a resin dice sold by the same maker also showed to be similarly unbalanced. Furthermore, the dice were unbalanced, but not weighted towards a specific side, so it cannot be attributed to a notable unevenness in the material during production. This occurred with both dice from the seller tested, so it cannot be concluded whether this would be a flaw in all liquid core dice, or just a flaw in their production process. Further tests on more dice from a range of sellers would be required to draw more confident conclusions as to the balance of liquid core dice. From the results of this experiment, it appears that using a liquid core dice would certainly make your TTRPG more interesting, however it is unlikely to lead to a particularly balanced game!

## References

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## Appendix 1 – Full Method Details

- The dice being tested had not been used before the testing, other than a couple of initial rolls upon delivery, so there is no risk of them having been damaged from previous use. This would be unlikely to be a significant issue, as dice are designed to be rolled repeatedly (though maybe not as intensely as in this experiment!) and not take damage, but it was noted anyway.
- I was careful, when rolling the dice, to not use any unnecessary force to try and minimise any damage from use throughout the course of the experiment; again this would be unlikely to be an issue, and if any damage were to be sustained it would be from bouncing through the dice tower rather than from my rolling, but it was also still noted. I attempted to balance this with still allowing it to fall a short distance in to the tower, to allow the randomness of the dice tower to still have full effect.
- When I rolled the dice each time, I picked it up from its previous roll and put it directly back into the top of the tower. This was to ensure there would be no accidental bias in the way the dice falls by rotating it before it was rolled through the tower.
- Whenever the result on the dice was a 6, 9, 1 or 7, I was careful to reorient the dice to ensure I was reading the number correctly, as there was a risk of mistaking the numbers for each other.
- After each prolonged break of rolling, I was careful to shake and rotate the liquid core dice before continuing, to ensure the core was not settled to the bottom of the dice. It would be expected that the liquid core would be designed to be the same density throughout the design to ensure the balance, but this was taken as a precaution just in case.
- If, when rolling the dice, it bounced out of the tray at the bottom of the tower, the roll was not counted and the dice was rerolled. This is common practice in TTRPGs [8], an accepted house rule for the sake of sportsmanship in dice games to make the natural result of the roll clear to the other players, so I followed it for the purposes of this experiment.
- The dice tower used in the experiment was a folding design for ease of doing the experiment. The tower was also transported carefully, and the dice were wrapped up to make sure they didn't get knocked or damaged.

Appendix 2 - Full Results Table of Liquid Core Dice Results

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Doubles
20		2	1	1		3		2		3	2	2				1	1		1	1	1
40	2	1			2		2	2	1	1	1	1	1		1	2	1			2	
60			1		1		1	1				2	2	3		3	3		1	2	2
80	2	1	2			2	1	1			1		1	1	2		1	2	2	1	
100	1	3			1		1		1		2		1	4	2		2		2		1
120				1	1	2			1		1	1	3	3	1	1			1	4	1
140	1			1	1		3		1	3				3	1	2	1	2	1		1
160	1		3	1				2	1	1	1	1	2	2			3	1	1		1
180	3		1	2	1	1	2		2	3				1				2	1	1	1
200	4	2	1	1	1	3	1				1	1		1			1		3		1
220					1	2	1	2		2			2	3	1		1	1	3	1	2
240		1		2	1	2			3	1	2	1		1	1		1	2	2		1
260	2	1		3			1	1	2	1		2					1	2	3	1	(19*3)
280	3	2		1	2	2			1	1	2			1	2	1		1		1	1
300		1	1	1	1			1		1	2		1	1		1	3	1	3	2	2
320	2		1	1		1	2	1	1	1		1		1	2	2	1	1	1	1	
340	2						2	1	1	1		1	4				3	1	2	2	1
360	1		2		1	2	1	1	1	2		2			1		1	2	1	2	1
380	1			1	2		1		1				1	1	1	3	1	2	1	4	1
400		1	2	1	1		2			2	3	2		1	2	1		2			
420		1		1	1	1		1	2		2		2		2	2	2	2		1	
440	1	1	2			2		2			1		4	1			2	1	2	1	2
460			2		3	2	1	1	1	1		2	1	1		3	1		1		1
480	1	1	3	2	2	1	1		1				3	1		1		2		1	1
500	1		2	2	1	2	2		1		2	2	1				1	2	1		1
520		2	1	1	2		2	1	1	1	2		1		2		1	1	1	1	
540		1	1	1	2	2		1			1	1		2	1	3	2	2			1
560	2			2	1					1		2	2	3			1	1	2	3	1
580	1		1	2	1		1	2		4	1		2			1		1	1	2	1
600	1	1	2			1	3	1	2	2				2	1		1		1	2	
620	4	1	2	1	2						2				2	1	1	2	2		1
640	2	2		2		2	2	1			2		1		1			2	1	2	2
660	3	1	2		1	2	1			1		1		1			1	1	1	4	2
680	1	1	1	1		1			1	2	2		1	2		1	2	2		2	
700		1	1	1	2	1	2	2		2	1	1	2		1			2	1		1
720	2		1	2		1	1	2		2	2	2	1			1	2		1		1
740	3	3	1	1		1		3	1	1	1	1			1			1		2	1
760	3	3			1	1	1			2	1				1	1	2	3		1	1
780	2	1			2			2			1	1	2		2	1	2	1	2	1	
800	2	1		1	3		1	1		1		1	1	1	1	1	2	2		1	
820	1			2		2	1	2		1	1			2	2	2	1		2	1	
840	1	1	2	1	3	1		1		1	2	2					1		2	2	2
860	2		1	3		1	1	2	1	1			1	1		1	1	2	2		1
880	2	1		2	1		2			1		1	1		4	1	2	1	1		1
900	4	1			1	2	2	1		1	1	1	1	3				1	1		1
920	2	1			1	1				2	1	2	1		1	2	5	1			1
940		1		1	1	1	1	1			1	4	1	1	1		4			2	(12*3)



1960					1	1	1	2	2	1	1	2	1	1	1	3	3	2	
1980	2	2	2		1		1	1		2	1	1	1		4	1	1	1	
2000	1	1	1	1	2		1	1	3		1	1			2	3	2	1	
2020	1		1	1	1		2		4	1			2		1		2	4	1
2040	1	1	1	1		1	1	2	1		1	1		1	1	2	2	3	1
2060	1		1	1	4				1	1	1	1	2	1	1	2	1	1	1
2080	1		2			1	3		1	2	3	1		1		2		3	4
2100	1			4		2	2	1	1	2	1	2	2				1	1	1
2120	1	2	1	1		1	2			3	1		1	1	2	1	1	2	
2140	3				1					1	1	2	2	2	1	1	2	1	1
2160	1			1	1		2	2	1	1	1	1	2	2	1	1	1	1	
2180	2		2	2	1		1			2	1	2	1	1	1	2		1	1
2200	2	2	1		2	2		1	1	1	1		2	1		1	1	3	
2220	1	1	2		2			2	1	3	2	1		1		1		3	1
2240				1	2			1	2	2	2	1	2	3		1	1	2	2
2260	2		1			1		2	2		1	1		2	3	1	2	2	
2280	3	2	2		1		1			1	1	3	1	1			4		
2300	2	1	1		2		4	1		2	1	1	1	1	1		2	1	1
2320	2					1		2	2	1	2		1	4		1	1	1	2
2340	2	3	4		1				1		2	1	1		1	2	1	1	
2360	1	1	1	1	1	1		1	2		2	2		1		3	2	1	
2380	1	1	1		1	1	2		4	2	1	1		1	3			1	2
2400	2	1	3		1	3	1	2			2		1	1	2	1			1
2420	2	2	2					2	2		1	1		2	2		2	1	1
2440	1	1	2	1				2	1	1		2	1	1		1	4	1	1
2460	2			1		1	1	2	2		2	1	1	3		2	1	1	(18*3)
2480	3	3			2	2		1		1	1	2	1	1		1	1		1
2500	5	2	1		1			3	2	1	2	1		1		1			2
2520	1	2	1	1	1	1	1			2	1	1		3		1		2	2
2540	3	1	1	1	2	1			4	1				3		1	1		1 + (1*3)
2560	1	1			2	1	4	2	1		2		2	1		1		2	1
2580	1		1	1	2		4	1	1		1			2		1	2	3	1
2600		1	1		1	1		1	2	1	1	1	1	1	2	3		1	2
2620		1	1			2	2	3	1		1	1		1	3		1	1	1
2640		1	1	2			1	1	1	2	1			1		1	1	2	3
2660	2	1	1		3			2		3	2	2		1			1	1	1
2680	1		1	1	3			1		3	2	3	2	2		1			2
2700	1	2		2	1	1	1			1	4	1			1	2	1	1	1
2720	1	1	1	3	3		1			2				1	1	1	1	1	3
2740	2	1		1	2	1			1		3	1	2		1		2	2	1
2760	1			2		2		1	2	2	1		1	2	1			3	2
2780	1	1			4	1	1	1	2	1	1	2	3			1		1	
2800	2				3		1	1	2	2			2		1		2	1	2
2820	1		1	1	2		2	2	1		1	4	1	2		1		1	
2840	4			1			1	4			2	2		1	1			2	2
2860	1	1	2	1	3	1					1	1	2	1		2	1	3	1
2880	2		3	1	2		2	2	1		2	1	1		1	1		1	1
2900	1	5		1	1	2	1	2	1	1	2				1	1		1	1
2920	2	1		3		2		1		1		1	1	2	2	1		1	2
2940	1	2	1		1	1	1		4	1		1	1		1	2	1		2

2960	1				2		1	2	1	1	1	2			2	1	1	2	2	1		1
2980	1	1	1	2		1	2		1	1					2	2	1	1	3	1		
3000	1	1		2	2		2	1	1			1	3		2	2		1	1	2		1
3020	3				1	1	4	1		1				2	1	1	3		1	1		1
3040	1	2		2	2	1	2	1	1		1		1				2		1	3		
3060	1	1	3			3				1		2	1	2		2		2		2		3
3080	2	2	2		3				2		1	1	2	1		1		1	1	1		2
3100	1	1	1	2	2		1	2	1	1		1	1	2			2		2	2		1
3120	2	1	2	1		1		1	1	1	2		1	2	1	1		1	1	1		
3140	1	1	1	1			2		1	1	1	3	2		1	1		2		2		
3160			1	1	4	1	2	1	3	1	2	1	1				2					1
3180	1	2	2			2	3		1		1	1	1	1	1			1		3		
3200	1	1	1		2		1	2	1	2				2	1		1	1	2	2		
3220	1		2	2	2		1	2	2			1	3				1	1	1	1		2
3240	1	1			2	2	1	1	1		2	2	1	4			1		1			1
3260	1	2		1	3		2	1			2			1	2	1	1			3		1
3280	1		1	1	2	1					2	2	1	2	2		2	1	1	1		
3300			2	1	1	1	2	3	1	1	2			2	1			1	1	1		
3320	1		2	2	1		2	2	1	1	3				1		2	1	1			
3340	2		3		1	1	1	2				2	1				4	1		2		
3360	2	3			1	1	2			1	1	2	1		1	1			1	3		
3380	1	2	1	1		2	1	2			1	1	1	1	2			3		1		
3400	1	2		1	1		2	3	1			1		1		2	2	1	1	1		
3420	2	1			1	1	1	1	2	2		1		1	3				1	3	2 + (20*3)	
3440	1	1	2	3	1	1	1		1	1		1		2	1	1	1		2	1		1
3460	1	1	1		1		1	3	3	1		1	1	1			1	1	2	1		1
3480	2	1	1			2	1	1		1	2	1	4		2		2					1
3500	1	1	1			1		3	2		2	2	3	1	1					2		
3520	1		1	1	2		1		1	1	1	2	1	2	2	1	1		1	1		2
3540		2			3	1	1	1	2	1			2	2	1	1	1		1	1		1
3560	1	2		1	1	1	4				1	1	1	1			2	1	1	2		3
3580	1	2			2		1		2	3	1	2		1			1	1	1	2		3
3600	1	1	2	2	2		2		1		2		4		1			2				
3620	1		1		4	1	3	2	2					3				1	1	1		3
3640		4		2	2			2	1	2	1	2			1	1	1	1				1
3660	3		1			1	2	3		1	1	1	2	1	1	2			1			1
3680	1	1				2		1	1	1	1	1	3	1	2	1			1	3		
3700	1	1		2	2		1	2	1		1			1	2	1	1	2	2			
3720	1	1	2	2	2	1				1		1		2	1	1	3		1	1		
3740	1		4		1	2	1		1	2	1	2	1	1			1			2		
3760		1			2	2	2	1		1	2	5	1	1			1			1		
3780				2	1	1	1	1	2		2	3	1	2	1		2	1	1			1
3800			2		4			3	3			1		1	1	2			3			1
3820				1		2	1	2	1	2	1	1	4	1		1			1	2		1
3840	2	3		1			1	1		1	2		2	1	1		2	1	1	1		
3860		1	1		2			2	2	1	1	1	1	1	2		1	1		3		
3880	1		1	1	1		1		1	1	2			3	2		2		1	3		
3900	2				1	1	4	1		1	2	2		1	1	2	1	1	1			1
3920	2	1	2	1	1		1	3			3	1					2	1	1	1		
3940	1		2		1		2		2	2	2	2	1		1	2	1		1			



3960	1		1		2	1		2	1	5		1	1		1		2	1		1	(10*3)
3980	1				1	2	1	2		1	1	2	1	2	1		1	2	2		1
4000	1		1	1	3		2	1			1		4		1		2	2			3
	262	187	184	172	235	167	222	219	175	184	194	204	205	190	195	126	209	206	209	255	

Appendix 3 - Full Results Table of Resin Dice Results

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Doubles
20		1	2		2	1	2	1		2	1		1		1	1	1	1	2	1	
40	1					1			1	1	2	2	3	1	2	1			3	2	
60		1	1				3	2	2		2			1	3		1		2	2	1
80	1	1	1	1	1		1		5			1		1	2	1		2	1	1	1 + (9*3)
100	3	3		2		2	1		3				1		1	1				3	2
120	1	1		1	1		1	1		4	2	2	2	1	1		2				
140				1					2	1	2	1	3	2		2	1	2	1	2	
160	4	2	1	1	2				1	1	2				1	1			2	2	1
180		1	2	1	2	1			1		1		4				1	2		4	1
200	1	1	1	1	2	1	1			3				1	1		1	1	1	4	1
220	1	1			2	2		3	1	1	4	1		2		1			1		2
240								3			2	1	3	3	2		1	2	1	2	4
260		2	1		1	1	1	2	2		1		1		1	1			3	3	2
280		1	1			1	3	1	1	2		1	1	2	2		1			3	1
300	1	1	1	1	1				1	2		1	1	2	5		1		1	1	1
320	1	1	1		1				2		1		3	2	1		2		2	3	
340	1	2			1		2		3	1	1				3		1	1	3	1	
360	1	3	3	2						1			1		1		2	1	3	2	1 + (19*3)
380	2		1			1	3			2		1	1	3		1			2	3	1
400	2	1		1	1					1	1	3	3	1					6		1 + (20*3)
420	1	1	3	1	1		2		1	1	1	1		1	1		2		1	2	1
440	1			1	2					1	3	1	1	1	1	2		1	3	2	2 + (11*3)
460	2	2		3			1	3	1	1	1		1	1	2	1		1			1
480	1	1	2				1	1	5			2	1		2	1			1	2	1
500	4		1		1	1	1	2	1		1			2	1	1	1		1	2	
520	1	1			2	1			1		2	2	2	3	1	1		1		2	1
540	2	1	1		2		1	1	1	2	1	2			1	2	2		1		1
560		2	1	2	1		2	1				1	3	2		1	1	1	2		1
580	1				1	1	1	1	1	1	2		3		1		3		1	3	1
600		1	2				1		2	1	1	1	1	3	2	1	2	1		1	
620	1		1	1	2		1		2		1		3			1	2		2	3	
640	2					3		2					4		3	1	1	1	1	2	
660	2	2	1	2	1	1				2		1	3	2					3		
680	2		1	1				1	2	1	2		3	1		1	1	1		3	2
700	1			4				1	3	1	2		1		1	1	1	2		2	
720	1	2	1			2		1	1	1	1	1	1	1	1	1	1	2	1	2	1
740	1	2	1		2		1		1	2	1	2	2	1	1				2	1	1
760	1	4			2		2		1		1	1		3	2	1				2	
780		2	1				1	1		1	1		1	3	1		1	3	1	3	
800	2	2	3	1	1	1	1	1	1			2	1				1	1		2	1
820		3		1		4				1			1			1	1	2	3	3	
840	1	2	2	2	2		1	1	3		3	1					1		1		
860	1		1		1	2	1	1		1	4			1	1	2	1	2	1		
880	1	1		2	4		3			1	2		1	1	1	1	1	1			1
900				1	1	1			1	1	1		2	2	2	2	2		4		2 + (20*3)
920	2	2		1				1		1		1	1	1	2	1	2	3	1	1	
940			1	1	1		1	2	1	1	1	2	2	2	2	1			1	1	2

960	2	2	1	1	1	1	1	1	2		1	1	1		2		2		2		2
980	1	1		1		1	3			1	1	1	2	3	1	2	1	1	1		1
1000	1		2		2	1		1	1	2	3		1		1		1	4		1	1
1020	1		2	1	1	5	1					2	2	2		1	1	1			1
1040		2			1	3	2	2	1	1		2		2	2	2		2			
1060	2	2					1	1		1	2	1	3	2	2	2	2	3		1	1
1080	1			1				1		1	1	1	2	2	2	1	2	2	2	1	
1100		2	1	1				1	1		1		2	1	3	1	1	2	1	2	
1120	3		1			1	1		3	2			1	1		1	1	1	4		1
1140	2	1	3				1		2		1	1	1	2		1	1	1	2	1	
1160					1		1	1		2	4	1	1		1	2	1	1	3	1	1
1180	1	1	4					3	1	3		3	2			1			1	3 + (11*3)	
1200	1	3	1	1		1		1	1	2			3	2			1	3		2	2
1220	2	1		1	2	2	1	1	1	1		1	2	1	1		1		2		
1240	2			2				1	2		4		3	2				1	3		2
1260			1	1	3	1		1	1	2	1	1	3	1		1	1	1	1		
1280			2		1	2	3	2	1	1	1	1	1	1	1	1	1	1	1	1	1
1300			1	3	3	1	1	1			1		2	1	1		1	2	2		1
1320	1			1	2		3	3	1	1		1	1	1	1			2	2		
1340	1	4		2		1		2		1	1	1	2	1	2		1	1	1		1
1360		1	2	1	2		2		1		2	3		1	1	1		1	2		1
1380		1	3			1	1	1			3	3	1		1	2	1	1	1		
1400	1				2	2		2	1	1	2	2		3			2	2			
1420		2	2		2		2	2		1	1		3					1	4		1
1440	1	3	1	1			2	1				2		2	1		1	1	4	1 + (20*3)	
1460	3	2	2	1	2		1		1	1		1		1	2	1		1			
1480	3	3			1		1	2	1		1	2		2	1	1	1	1			
1500	2	1	2		1	1		3	1		3	1	2		1	1	1	1		1	1
1520	2		1					1	2		2		4		1	1	1	1	3	3	1
1540				1	2	2		2	3	2				1		1		1	5		3
1560		1	1				1	1	2		3	1	1	1	2	2	1	1	1		
1580	2	1	1		2	1		1		1	1	1	1	2	1		3	1	1		1
1600	1			1	1		1			2	1	4	1	1		3	2		2		1
1620		2		1		2	3	2	1	3	4					2					3
1640		1		2	2		2	1	1	1				1	2	3	1	1	2		1
1660	1	1		1	1				3			1	1	3	1	2	2	1	2		1
1680	1	1	1			1	2		2		4		1	2	1	1	1	1	1		
1700	1	1			2	1	3	1	2	1	3	1			1	1		2			1
1720	2	1	2			2	3	1			3		2	1		1	1		1		
1740		1	1	1	1	1		1		4		2	1	4	1			1	1		
1760	2	1	1		1	1				1	4	4	2					1	2		1
1780	3		2	1			1			2		4	4			1		1	1		1
1800			2	1	1	1		1	1	1		2	2		2	1	1	3			
1820		2			1	2			3	1		4			2			3	2	(13*3)	
1840	1	2	2	1	1			1		1	1	1	2		1		1	2	3		1
1860			1	1		2	1	2			1		3	1	4		1	1	2		1
1880		2	1			1	1	1		1	1			2	1	1		5	3		2
1900	2	2		1	2		1		1	4	1	2	1	1				1	1		
1920		1	1	1	1	2	1	1	1	1	1		1	1		1	1	1	4		1
1940		2					3		2	1	3	1	2	1		3		2			1

1960	2	1		1	2	1	2			1	1		1	2	1	2			2	
1980	2	2	1		1	1			1	2	2		1	1	1	1	1	1	3	1
2000		1		1		3	1	1	1	1	1		2	3	2	1	1	1	1	1
2020		1	1		1			1	1	1	2	2	1	2	2	1	3	1	1	1
2040	4		1		1		1	1		3	1		2		2		4	2	1	
2060	1	3	1		1		1		3		1	3		2		1	1	2	(9*3)	
2080			1		2	1		1	1	1	2	3	2		2	1		3	2	
2100	2		1	1	1		2	1	1		1	1	2		2		2	1	2	
2120		2				1	1	1	3		1	2	1	1	1	2	1	2	1	
2140	4	3					1		1	2	2		1		1		1	4	4	
2160	1			2		1	1	1	1	2	2	2	1	1		1	1	2	1	
2180	1		1	2	1	2	1	1	1	4	1	1	1				2	1	1	
2200		1		2	2	2	1				1		1	4	1	2	1	2		
2220	2			2	1	1	4	1	1	1	1	2		1		1	1	1	3	
2240			3	1	1		1	2	1		3	1			1	2	1	1	2	
2260	5			1		1	1	1	1	1	1		2		1	2	1	2		
2280	2	2	2	2	1		3	2		1		1		1	1	1	1	1		
2300	2	1					1	3	1	2	1		2	3		1		3	2	
2320	1	3	1					1	2	1	1			2		2		3	3	
2340			2		1	1		1			1	3	3	1		2	1	1	3	
2360	1		1	1		2	3	1	3		1	1		2			1	3	1	
2380		1					3	2	1	2	1	2	1	1	3		1	1	1	
2400	3			2	2		1	1	1		1		1	2			2	4		
2420	1	1	3		1		1	1		2	2	1	1	1	2	1	1		1	
2440	1	3			2		1		2		3		1			2	1	1	3	
2460	2			1	1	1		1	2	2	1	1			2	1	2	1	2	
2480	2	1		1	1	2		3	3				1		2	1		2	1	
2500	2	2	1	1		1	2				1	2	1	2			1	2	2	
2520	3				2	1	1		2	1	1	2	1	1	1		3	1		
2540	2	2	2				1		2	1		3	1		1			5	3	
2560		2						1	1	2	1	1	1		1	3	3	1	3	
2580	1		2			2			3	1		1	3				1	2	4	
2600	2		3		2		1	1		1		1	2		2		1	4	1	
2620		3		2		2	1		1			1		2	1	1	3	2	1	
2640		1	1	1			1	1	1		4		2	1		1	1	2	3	
2660	1	3			1	1			3	2	1	1	1		1	1	2	2		
2680	2		2	1					1		1			2	1	1	1	3	5	
2700	1	1	5		1		1	1		1	1	2	2		1		1	1	1	
2720	1		3		2					2	1	3		1	1	2	1	1	2	
2740	1		2		1	2	2		2		1	2		1			4	2	4	
2760		2	2			1	4		1		1		1			3	2	2	1	
2780	1	1	2	2	2	1		1	1	1				4			1	3	1	
2800	1	1	1	1			2	1	1	1	1	4		1		2	1	2	1	
2820		1	3		1	1			1	2	2		1	1	2	2		1	2	
2840		3	3	1		1			1	1	1	2	2	1	1		2	2	1	
2860			1	1		1	3		1	1			2	2	2	2	1	1	2	
2880	3	3	1					1	1	2			3	2	1		1	2	2	
2900	2	1	2		1			1				1	1	1	1	3	2	1	3	
2920	1	1				1		3		1	1	2	2		1	1	3	2	1	
2940		1	2	2			1	1		2	1		1	2	1	1		4		

2960	1			2	1	1	2		2	2	1	2		1	1		2	1	1	1
2980	3		4	1			2	1	1		1	1	1		2		1	2	1 + (3*3)	1
3000	1		1	4	1			2			4		2		1	2	1	1	1	1
3020		3	1		1	1	3		1		3	2	2	1			1	1		2
3040	1	1	2	1					2	1	2	2	1	2			2	2	1	1
3060	2		2	2	2			1	1		2		1	1			1	1	4	1
3080	1	1	4	2	2			2		2	1	2	2						1	1
3100	1	1	1		1	1		1	1		2			1	1	4		1	4	3
3120	1	1	2		1	1	1	1	1		1			4		1	1	1	2	1
3140	1	2			1	1	1	1		1		1	1	3	3	2			2	
3160	1	1		1	1		2	1	1		1	2		1	1		2	1	5	2
3180	1	2	1	1	1	2	2	1	1	1	1	1		1	1		2	1		
3200		1			3	1		2	2	1				1	1	2		2	4	1
3220			1		1		2	2			1	2	3	2	1	1			4	3
3240			3	1	2		2	1	1		2	1	1	1	1		1	2	2	1
3260	2	2		1	2		1		2		2		1	1	1		2		3	
3280	3	1			1	2		1	1	1	2	2		1	1		2	1	1	1
3300	1	1	1		1	1		1		2	3			1	1	2	1		2	2
3320	2	3	3	1	1	1		1	1		1	2	1	1					2	1
3340	1	1		1	2		2		1	1	2	1	1					2	5	1
3360		1		3	1		1		1		4	2	1	1			1	1	3	1
3380		1					3	1	2		3	1	3	1	1		1	1	2	1
3400					1	1	1		1	1	4			2	2	1	1	3	2	2
3420	2	1	1	1	1	1	1	2		1			1	2	2	1			1	2
3440	2	1		2	1	1	2		2	1	1	2		1	1		1	1	2	2
3460			1		4	1	1	2	1	1	1	1	1	1	1			2	2	2
3480	1			1	3				2		1				3		2	3	4	1
3500	1		1	1	2		1	1		2	1	2	2	1				4	1	
3520		2	1		1	1			2	2	1	1		3	3			1	2	1
3540	1	3	1	1	2	1	1	2	1		1	1		1			1	1	2	
3560	1			1	1	1	2	1	1	1	1	2	2		1	1		2	2	
3580	1	2		2	1	1	2	2		1	1		2	2	1			2	1	1
3600		2	1				1	2		2	1		2	1		1	1	2	2	2
3620		3	4		4	1		1	1		1	1			2		1	1		2
3640	1	2		1	2	1	1	1	1		1	1	1	2			1	1	1	2
3660	2		1		3	1	1	2		2	2		1	1		1	1	1	1	
3680	2	1	2		1		3	1		1	1		1	1	1		3		1	1
3700	2	3			1	1	1	1	1				1	2	2	2	1	1	2	1
3720	1	1	2	1		2			1	2	1		2	1	2	1		1	2	
3740	1	2	1		4	2	1		1	1			1	2		2		2		1
3760	2		1	1	3		2	1			1	1	2		1	2		1	2	
3780	1		2		1		1		1	3	1	3		3	1		1		2	1
3800	1	1	1		2	2		1		1		1		2	1	2		2	3	
3820	2			2	2	1	1		1		2	1	2	1		1	1	2	1	
3840	1	1	2		1	1		2	1			2		1	2	1		1	4	
3860	1	1	2	1	2	1	2	2	1		1	2		1	1		1	1	1	
3880	1	2	1		2	1		2	1		1	1	1	1	1	1		2	1	
3900			2		2		1	1		1	2	1	1	1	1		1	2	3	1
3920	4		2			3	2		1	2	1				1	1		1		2
3940	2	1	2	1	1	2		2		5				1	2				1	

3960	1	2	1	1				1		1			3			2	1	2	5	2	
3980	3		1			2	1		2	1	1	3	1			2	2		1		
4000	3	1	2	1		1	2						1	2		2	2		1	2	1
	226	217	205	136	194	152	197	165	182	170	227	161	260	193	229	139	182	147	238	380	