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What is the smallest planet where an astronaut could accidentally escape gravity?

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Abstract
In this article, it is evaluated what the required dimensions of a small object in space (such as a small planet, a moon or an asteroid) should be in order to have a gravitational pull that is just strong enough to not let humans escape it by running and jumping. By evaluating the escape velocity, it is found that a mass-to-radius ratio of 5.8×10^{10} kgm^{-1} will prevent regular astronauts from escaping, and a ratio of 1.15×10^{12} kg m^{-1} will prevent even the fastest human alive from escaping the gravitational pull. Objects in our solar system that are near these ratios are Leda, 433 Eros and S/2003 (130) 1.

Introduction
Visiting planets other than our own has always sparked humanity's imagination, not only in a scientific way but also in for example (children's) literature, with books such as *Le Petit Prince* by Antoine de Saint-Exupéry [1, 2]. In it, the little prince visits a number of planets which by the illustrations of the author himself appear to be very small. With humanity's reach stretching further and further into the universe and NASA sending expeditions to not only planets but asteroids as well, one might wonder: would it be realistic for man to one day wander about on these tiny space-rocks? And should these astronauts then be careful not to accidentally run to fast or jump to high, launching themselves into the black vacuum of space?

Escape velocity
To understand why we aren't able to, say, throw a ball to the moon, we need to take a look at one of gravity's many aspects: gravitational potential energy U. The energy can (very roughly) be compared to energy required to stretch a spring. The equation is as follows:

$$U = -\frac{GMm}{r}, \tag{1}$$

where G is the gravitation constant, m the mass of one object (usually the smaller one, like a rocket), M the other mass (usually the larger one, like the planet) and r the distance between the two centres of mass. If we consider how much energy must be paid to move an object from the surface of the planet with radius R_P to infinity (so beyond the grasp of gravity), we must evaluate the difference in gravitational potential energy between the two situations. This looks as follows:

$$\Delta E = U_1 - U_0 = -\frac{GMm}{\infty} + \frac{GMm}{R_P} = \frac{GMm}{R_P}. \tag{2}$$

This means that an object that has at least that much kinetic energy, can escape the planet's gravity from its surface. Only at infinity will all kinetic energy be transformed into gravitational potential energy and cause the object to come to a stop and start to fall, which means that this will not happen and the object escapes. We can compare the result from equation 2 to the equation for kinetic energy:

$$\Delta E = \frac{GMm}{R_P} = E_{kin} \equiv \frac{1}{2}mv^2. \tag{3}$$

With a little rearranging, this gives:

$$v_{max} = \sqrt{\frac{2GM}{R_P}}, \tag{4}$$

with v_{ma} the maximum velocity for anything on the surface of the planet that you do not want to escape gravity, also known as the *escape velocity.* For the sake of calculability, all asteroids and planets are assumed spherical.

How slow should you go?

Of course, no one knows who will be sent to space in the future. Also, it is difficult to make an estimation of the preferred 'wandering-about'-velocity of astronauts in vacuum, as this is probably a highly personal matter. Therefore, perhaps it is best to play it safe and to make sure that even the fastest man alive will in the worst case only launch himself into a *nearly* infinitely large orbit around the planet/asteroid and not irreversibly drift off into the void. The fastest recorded running velocity of a human is at the moment 44.72 km hr^{-1}, or 12.42 ms^{-1}, set by Usain Bolt [3]. We will take this as the minimum escape velocity as we assume that wearing a space suit will hinder ambitious astronauts enough to not break the world record. Looking at equation 4, this gives a minimal ratio for M/R_P of 1.15×10^{12} kgm^{-1}. Graphically, that looks like this:

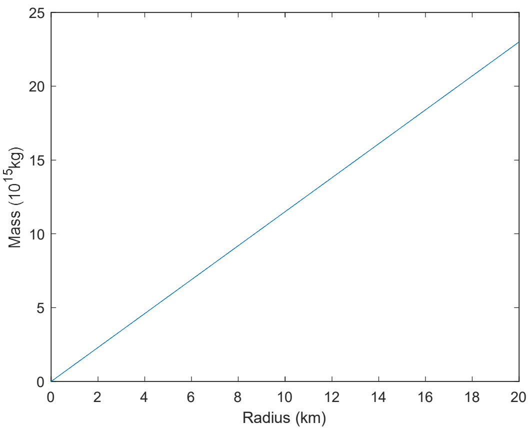

Figure 1 – Graphic representation of required mass/radius ratio.

Everything above the line is safe. Underneath the line are asteroids and planets where Usain Bolt might escape from.

Known asteroids

Above ratio provides a very convenient way to quickly assess whether or not someone might be able to escape an asteroid's gravity. From going through databases, objects with a radius under about 8 km seem not to satisfy the ratio. However, there are small bodies within our solar system that do. One of them is Jupiter's moon *Leda*. It has a mean radius of about 10 km, and a mass of 11×10^{15} kg. It has an escape velocity of 12 ms^{-1} (or 43.2 km hr^{-1}) [4], so a regular astronaut does not have to worry about flying off and even Usain Bolt will have a hard time reaching this.

Another asteroid close to the threshold, but a little more below it, is *433 Eros*. It has a mean radius of 8.4 km and a mass of 6.687×10^{15} kg. That means that the escape velocity at its surface is just over 10.3 ms^{-1} (37.1 km hr^{-1}) [5], which means that if the astronaut does not decide to go running, he is probably safe.

If astronauts are confident that they will not exceed 10 km hr^{-1}, an M/R_P ratio of 5.8×10^{10} will suffice. These are more abundant: the smallest object in our solar system that satisfies this is *S/2003 (130) 1,* a moon of asteroid *130 Elektra* that has a radius of about 3.1 km. At the surface, it has an escape velocity of approximately 14 km hr^{-1} [6].

Conclusion

For objects within our solar system, the smallest space-object on which astronauts can walk around without having to worry about escaping its gravity, is actually a moon of an asteroid and has a radius of about 3.5 km. Known objects smaller than that have an escape velocity that is so low that astronauts would have to be very careful. If Usain Bolt were to try to set a new world speed record in space, he would have to do it on an asteroid or moon that has a radius larger than roughly 10 km, as he will float off into infinity otherwise.

Discussion

It should be noted that similar researches have been done [7, 8, 9, 10], however they were not consulted or considered for this paper and they operated either from a different perspective or had a different aim.

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