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When New York Freezes – An Exploration into the Ice Formation in the Movie 'The Day After Tomorrow'

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Abstract
This paper discusses whether the 'big freeze' caused by an extreme polar vortex in the movie '*The Day After Tomorrow*' would be possible. It was assumed that the water covering New York had similar behavioural characteristics to that of a shallow lake and that the width of ice formed was large enough to support a medium truck. Using re-arranged thermodynamic equations, it was concluded that temperatures dropping to -150˚F would not be enough to freeze the ice since it would take 45.66 hours. Since the ice froze overnight as the characters slept, this aspect of the movie is not plausible.

Introduction

The 2004 science-fiction disaster movie '*The Day After Tomorrow*' follows the story of a group of people experiencing extreme weather events in the light of climate change [1]. A melting glacier causes a 40-foot storm surge over New York which is subsequently frozen by an extreme polar vortex that drops the temperature to at least -150˚F [1]. This paper aims to calculate the time required for the ice to freeze and determine if such an event as portrayed in the movie is possible.

Deriving the Equation for Ice Freezing

The traditional method for predicting the thickness of the ice is based on a very simplified version of the Stefan problem [2]. The Stefan problem aims to describe the temperature distribution in a homogenous medium undergoing a phase change. This includes the phase change of water turning into ice, and specifically aims to determine the relationship between ice thickness and time [2].

The simplified version begins by expressing the heat flux through the ice in the form of equation 1:

$$Q_i = \frac{k(T_m - T_s)}{h}, \qquad (1)$$

where Q_i represents the heat flux through the ice, k is the thermal conductivity of the ice, T_m is the temperature of the water-ice interface, T_s is the temperature of the surface above the ice, and h is the thickness of the ice.

The heat flux through the top of the surface is balanced through production of the ice at the bottom of the surface [2]. This is represented by equation 2:

$$\rho L \frac{dh}{dt} = Q_i, \qquad (2)$$

where L represents the heat of fusion of ice, ρ represents the density of ice, and t represents time. The derivation of dh/dt shows the change of the ice width over time.

Equating equations 1 and 2 results in equation 3:

$$\rho L \frac{dt}{dt} = \frac{k(T_m - T_s)}{h}. \qquad (3)$$

If equation 3 is integrated with respect to dh and dt (assuming the initial conditions for each are equal to zero), the result is equation 4 which represents the relationship behind ice width and time.

$$h = \left(\frac{2k}{\rho L}\right)^{1/2} [(T_m - T_s)t]^{1/2}. \qquad (4)$$

To make the equation more reliable for real life conditions, this paper will use a variation of equation 4 which adds the 'Bulk Transfer Coefficient'

H_{ia}. This represents the proportionality constant between the heat flux and the thermodynamic driving force for the flow of heat (i.e. the temperature difference, ΔT). It was first proposed to be added to the traditional equation by George Ashton in 1989 [2]. Adding the constant yields equation 5:

$$h = \left[\left(\frac{2k}{\rho L} \right)^{1/2} (T_m - T_s)t + \left(\frac{k}{H_{ia}} \right)^2 \right]^{1/2} - \frac{k}{H_{ia}}. \quad (5)$$

The H_{ia} constant usually falls between 10 and 30 $Wm^{-2}K^{-1}$ where higher values are associated with windy conditions [2, 3].

Calculating the Freezing Time

This paper will use equation 5 to determine the amount of time it would take the extreme polar vortex in the movie to freeze the water over New York. To do so, the following assumptions must be made:

- The water flooding New York will have the same characteristics as that of a very shallow low salinity lake. This will avoid complicating the equation as ice formation in lakes is not affected by changing velocities and turbulences associated with ice formation in rivers [3]. As well, the water in the movie has time to settle to relative placidity before it is frozen [1].
- The properties of the ice will be consistent with that of ice formed with minimum impurities. Therefore, the values used for the density (ρ), heat of fusion (L), and the conductivity (k) of the ice will be 920 kgm^{-3}, 3.34×10^5 Jkg^{-1}, and 2.24 $Wm^{-1}K^{-1}$ respectively [2, 3].
- It is not stated or identifiable in the movie how much wind affects the water as the ice is forming. For this reason, the H_{ia} value of 20 $Wm^{-2}K^{-1}$ will be used. The value of 20 is the standard value to use when no wind data is available [3].
- It is impossible to determine how thick the ice that forms over New York is as there is no mention of it in the movie or on fan sites [1]. In promotional posters for the movie, it appears that ice is frozen to the waist of the Statue of Liberty [1]. Since the Statue of Liberty has a height of 305.12 feet, it can be assumed the ice reaches a height of half of this, or 152.56 feet [6]. This is an unrealistic aspect of the movie, as only 40 feet of water covered New York [1]. While snow accumulation could have eventually raised the apparent height of the ice, the thermodynamics of snow and ice are complex, and this aspect of the movie will be ignored in way for simplicity. As characters in the movie were able to walk on the ice without fear, this paper will assume that the ice was at least 15 inches (0.381 m) thick from the top. This is strong enough to support a medium sized truck without cracking [4, 5]. This width is likely much larger than what would be required for the characters to walk on it in the movie, but it is the largest width given on multiple ice safety websites for safe ice walking, and is used to balance out the fact the ice in the movie is higher than what is calculated [4, 5].
- T_m (the temperature of the water-ice interface) will be equal to 0˚C (273.15 K) as this is the standard freezing point of water [2, 3]. T_s (the temperature of the surface above the ice) will be represented by -150˚F (172.039 K) since in the movie, the temperature drops to at least -150˚F [1].

To determine the time that it would take the ice to freeze to a minimum of 0.381 m, equation 5 is rearranged to solve for t as represented in equation 6.

$$\frac{\left(h + \frac{k}{H_{ia}}\right)^2 - \left(\frac{k}{H_{ia}}\right)^2}{\left(\frac{2k}{pL}\right)^{1/2} (T_m - T_s)} = t. \quad (6)$$

Subbing the proposed values into equation 6 results in a calculated time of 1.65×10^5 s or 45.66 hours. Since in the movie the deep freeze occurred overnight as the characters slept, the calculated value of 46.55 hours shows that the scene is not accurate.

Conclusion

This paper aimed to calculate whether the movie '*The Day After Tomorrow*' was accurate in the time it took New York to freeze over. It was assumed the water froze to a width that was able to withstand the weight of a medium truck and that the water characteristics were like that of a shallow lake. With temperatures dropping to -150˚F as stated in the movie, the overnight freeze that occurred would not have been feasible if the characters slept overnight, since it would take 45.66 hours to freeze.

References

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