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How humans affect the Earth's rotation with aeroplanes

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Abstract
This paper looks at the effect that planes could have on the rotation speed of the Earth through the Earth's moment of inertia using the principle of conservation of angular momentum. It is determined that depending on whether the planes are empty or full the rotation speed of the Earth will increase between 15.887 and 29.504 pico seconds respectively.

Introduction
In 2016 there were over 3 million recorded flights involving UK airports [1], showing the extent to which humans have adapted to using flight as a common means of transport.

It is estimated that there are currently 39,000 commercial and military planes in service in the world, with Boeing predicting another 39,000 being required over the next 20 years [2]. This number does not include light aircraft and so the actual number is likely much higher.

The scale of this can be seen on the internet where planes can be tracked in real time from their transponders (figure 1). But what kind of impact does this have on our world? This paper looks at the potential impacts on the Earth's rotation if all of these planes were to take to the sky at the same time.

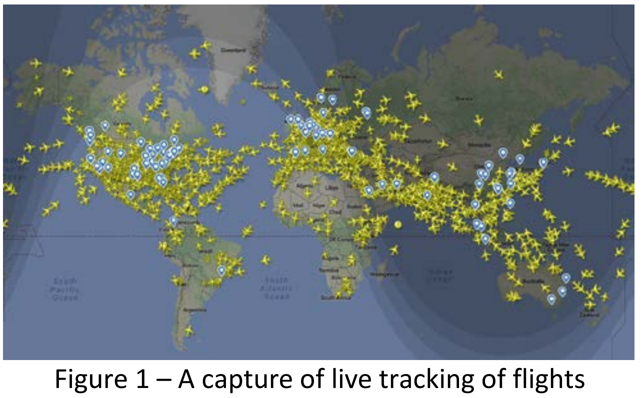

Figure 1 – A capture of live tracking of flights from their transponders. At the time of capture there were over 13,000 flights in the air [3].

Theory and Calculations
To calculate the change in the rotation speed the angular momentum of the Earth is to be calculated. The angular momentum of an object is given by:

$$L = I\omega \qquad (1)$$

Where I is the moment of inertia of the object and ω is the angular speed. For the model, the Earth and aeroplanes are considered an isolated system with no external forces acting on it. The Earth itself is modelled as a sphere and the planes are modelled as a hollow sphere, where they are all in flight at the same height and they are equally spread around the Earth, producing an equal distribution of mass (figure 2).

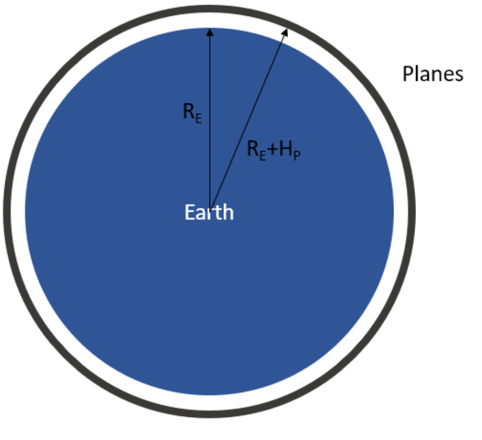

Figure 2 – A cross section of the model used, the Earth is a solid sphere with the planes a hollow sphere as shown by the black circle. It is assumed the planes are equally spread throughout the black area to produce an equal mass distribution.

As the Earth-plane system is considered to be isolated with no external forces acting and no net energy entering or leaving the system then it is assumed that momentum is conserved, and so the redistribution of weight outwards decreases the angular velocity. An example of this is an ice skater spinning, with their arms drawn in they rotate faster than with their arms extended.

For this model the moments of inertia for the two parts of the system are given by [4]:

$$I(Earth) = \frac{2}{5}MR^2 \quad (2)$$

$$I(Planes) = \frac{2}{3}MR^2 \quad (3)$$

With conservation of momentum the overall equation is:

$$I_1\omega_1 = I_2\omega_2 \quad (4)$$

Where I_1 is the moment of inertia for the Earth assuming all of the planes are on the ground and I_2 is the moment of inertia for the Earth and the planes flying. ω_1 is the normal speed of rotation of the Earth and ω_2 is the speed of rotation when the planes are in flight.

Rearranging the above gives the following equation that is used for the calculation:

$$\omega_2 = \frac{I_1}{I_2}\omega_1 \quad (5)$$

$$\omega_2 = \frac{\frac{2}{5}M_E R_E^2}{(\frac{2}{5}(M_E - M_P)R_E^2 + \frac{2}{3}M_P(R_E + H_P)^2)}\omega_1 \quad (6)$$

For these calculations the values used are as follows:

- 39,000 planes are all in flight at the same time.
- The planes are evenly spread around the globe.
- The planes all weigh the same amount and that they are all roughly the weight of the most common planes in service at the moment of 42000 kg empty and 78000 kg max [5]. $M_P = \text{mass of planes} \times \text{number in flight.}$
- The mass of the Earth is: M_E = 5.9722×10^{24} kg [6].
- The radius of the Earth is: R_E = 6378136 m [6]
- The planes fly at a height of: H_P = 10668 m [7]
- The angular speed of the Earth using a sidereal day is ω_1 = 7.292115853×10^{-5} rad s^{-1} [8]

As the change considered is so small Wolfram Alpha [9] was used as it was the only available software that produced the required precision. This found a value of I_1/I_2 = 0.99999999999999981562 to 20 d.p. for the empty planes.

This then produced a value of ω_2 = 7.2921158529999986555×10^{-5} rad s^{-1}. Which is a reduction of 1.3445×10^{-20} rad s^{-1}. Similarly for full planes I_1/I_2 = 0.99999999999999965758 to 20 d.p., with ω_2 = 7.2921158529999975031×10^{-5} rad s^{-1}, a reduction of 2.4969×10^{-20} rad s^{-1}.

These then correspond to an increase in day length of 1.5887×10^{-11} s or 15.887 pico seconds for empty planes and 2.9504×10^{-11} s or 29.504 pico seconds for full planes. These numbers are incredibly small, and would produce no real noticeable impact on human lives and activities. However, this change is reasonable considering the mass of the Earth compared to that of the planes.

The model is not a perfect model however, as the precision of the initial values used limits the accuracy of the final values produced. Also, due to the precision any actual differences between the model and reality would produce a much larger effect on the final result than would normally be expected.

Conclusion

The result does show that human activities with flight do affect the Earth, even if only in a negligible way. Potentially, with future improvements to the method of measuring an even more precise result could be calculated. Alternatively the principles of this model could be used to produce the number of planes humans would need to build and fly to make a noticeable difference.

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